International Journal of Electrical and Electronics Engineers ISSN- 2321-2055 (E)
http://www.arresearchpublication.com IJEEE, Volume 07, Issue 01, Jan- June 2015

REVIEW OF THE VARIOUS SPHERE DECODER APPROACHES FOR MIMO-OFDM SYSTEM

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ABSTRACT

Sphere decoding algorithms for Multiple Input Multiple Output (MIMO) - Orthogonal Frequency Division Multiplexing (OFDM) linear channels are considered. The principle of the sphere decoding algorithm is to search the closest lattice point to the received signal within a sphere radius, where every codeword is represented by a lattice point in a lattice field. In this paper, a comprehensive survey of existing sphere decoding algorithms is presented. The existing search strategies are described in a unified framework and their pros and cons have been described.

Keywords: Lattice theory, MIMO system, MIMO detection, OFDM, Wireless Channel

I. INTRODUCTION

The pursuit of high speed wireless data services has made the communication researchers relatively active. The limitations of wireless medium are a challenge to the researchers as demand is continuously increasing for available limited bandwidth. This has led to the search for a reliable and high data rate communication system. To fulfill these requirements, MIMO systems are preferred because they provide high data rate transmission over wireless channels and theoretically show considerably improved spectral efficiencies. In multi-antenna system, space-time (along with traditional error-correcting) codes are often employed at the transmitter to induce diversity. Moreover to secure the highly reliable data transmission, special attention should be given to the design of receiver antenna. The received signal is the combination of the transmitted signals affected by noise, Inter Symbol Interference (ISI) & Inter User Interference (IUI).

MIMO systems have attracted much attention for more than a decade because they provide high data rate transmission over wireless channels and theoretically show considerably improved spectral efficiencies. Optimal detection of signals or the Maximum Likelihood (ML) transmitted over MIMO channels is well-known to be an *NP-complete problem (NP-Complete (Non-deterministic polynomial-time complete) problem is a class of decision problems where a given solution can be verified, but there is no efficient way of locating that solution. Computational time increases rapidly with the problem size)*. An optimal performance can be obtained by implementing the ML decoder but its exponential complexity makes it unrealizable in practical systems when a large number of antennas and higher order modulation schemes are used.

In order to attain ML performance at reduced complexity, a multichannel equalizer is used to suppress ISI and IUI. By using nonlinear different equalizers like Vertical Bell Layered Space-Time (V-BLAST) [1] or linear equalizers like Zero-Forcing (ZF) equalizer [2]-[3] and Minimum Mean Squared Error Detector (MMSE) [4], better performance in terms of high signal to noise ratio (SNR) can be realized.

But a feasible option to obtain optimal performance in case of larger number of transmit antennas and/or higher modulation schemes is the application of Sphere Decoder (SD), whose computational complexity is independent of the total number of possible transmit vectors. SDs achieves similar performance to ML decoder with reasonable computational complexity. This is due to the fact that SD examine only the vector candidates which fall within a sphere of a given radius $\sqrt{C}$ centered at the received vector y, instead of examining the entire possible transmit vectors as shown in fig. 1.

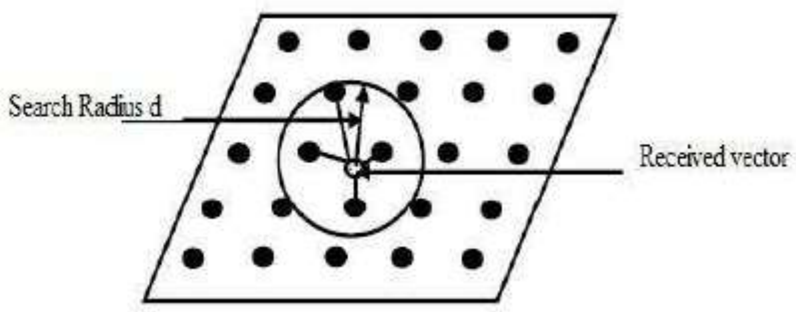

Fig.1: A 2- Dimensional Geometric Representation of Sphere Decoding [5]

II. THE MIMO MODEL

Consider the linear MIMO system as shown in fig.2 to communicate over the channel. We have to find the detection of a set of M transmitted symbols from a set of N observed signals.

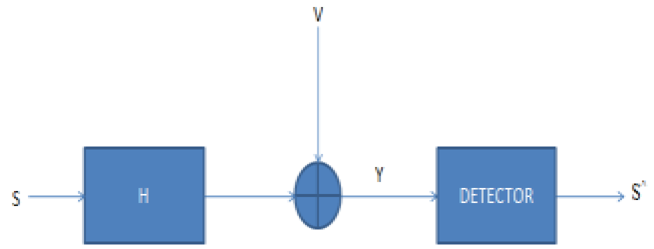

Fig.2: MIMO Communication system diagram [6]

MIMO channel is given below:

$y = Hs + v$ (1)

where $s \in S_m$ is the finite set of transmitted vector symbols , $y \in F_n$ is the received signal vector, $H \in F^{nxm}$ is the channel matrix and $v \in F^n$ is the additive white Gaussian noise. Here F is the set of real or complex numbers [6]. In the most general term, a detector or receiver refers to a mapping which takes the vector of received signal y and the channel matrix H as inputs and thus produces an estimated symbol vector, ŝ as output. That is, a detector is defined by some (possibly random) map.

$\phi: F^n \quad F^{nxm} \rightarrow s^m$ (2)

where $\hat{s} = \Phi (y, H)$ and F is real or complex. Computation of Φ relates to the implementation of the detector. The possibility that the minimum probability of error provided by the receiver in case of transmitted messages $s \in S^m$, is the ML receiver expressed as:

$||y - H\hat{s}||^2$ (3)

III. SD STRATEGIES

SD is an efficient strategy that computes all the lattice points within a sphere with a certain radius. This strategy is based on integer lattice theory. The concept behind the sphere decoding is to limit the count of possible code words by considering only those which are within a sphere centered at the received signal vector. This enumeration strategy was first introduced in digital communications by Viterbo and Biglieri.

The SD algorithm for Spatial Modulation (SM) MIMO systems has two types of searching strategies, the Fincke - Pohst (FP) and the Schnorr-Euchner (SE).

3.1 FP Strategy

The FP strategy is considered to be the original sphere decoding algorithm [7]. This method considers all hypotheses in natural order and the search is starting with the first one as shown in fig. 3.

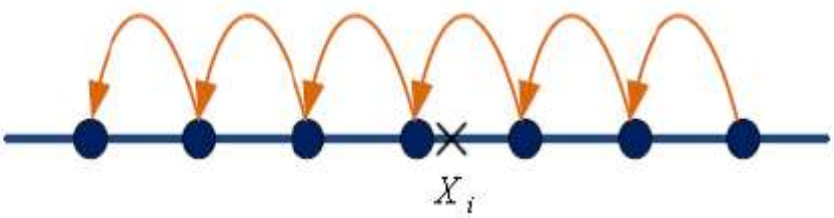

Fig.3: Fincke - Pohst Strategy [5]

An important characteristic of the FP strategy is that a search radius $\sqrt{C}$ must be specified. However, if C is too large, many lattice points will have to be computed and a large number of points may also be cancelled out. If it is too small, no lattice points will be found and then decoder must be restarted with a larger search radius. Both of these factors negatively impact the overall computation time and thus it is well-known that one of the main weaknesses of the FP decoder is the sensitivity of its performance to the choice of C. A typical choice is the distance to the Babai point (BP) [8].

A simplified SD algorithm is given below:

Step 1: Input y, C, H and S.

Step 2: Compute Gram matrix $G := H^T H$ and find QR decomposition $\{q_{ij}\} :=$ Q-Chol(G).

Step 3: Compute $\rho := H^{-1} y$

Step 4: Initialize $d^2 := C$, $T_n := C$, $S_n := \rho_n$, $i = n$

Step 5: Evaluate the followings:

$U_i :=$ Q-Up($\sqrt{(Ti/qii)} + S_i$, S);

$L_i :=$ Q-Low($-\sqrt{(Ti/qii)} + S_i$, S);

$N_i :=$len(L_i, U_i, S);

$y_i :=$find(L_i, U_i, S);

$z_i :=$ sort(y_i, U_i, S);

Step 6: Output $s_i = z_i(1)$

Step 7: Next i [9].

3.2 SE Strategy

Schnorr and Euchner introduced an algorithm which does not require an initial radius estimate [2]. It added a small but significant improvement to the FP approach. The FP strategy search for the valid nodes without any ordering, whereas in SE strategy, the valid nodes of each level are spanned in a zigzag order starting with the closest middle point as depicted in fig.4.

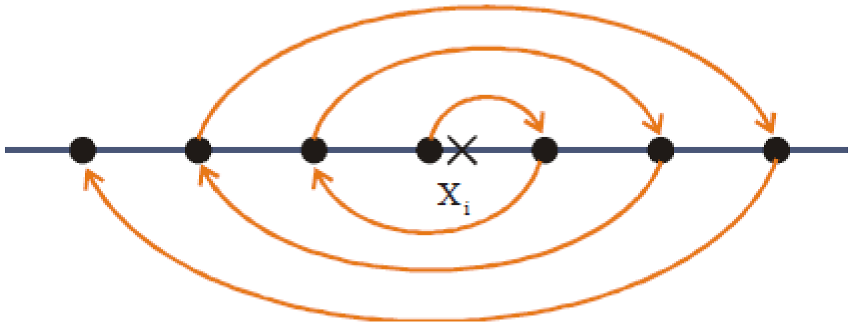

Fig.4: Schnorr-Euchner Strategy [5]

SE strategy considers symbol close to the BP solution and if a point is found, then the radius is updated (reduced) and so on. The SE enumeration is more efficient than FP and is less complex in terms of computations than FP. The tree is explored depth-first and child nodes at level i are prioritized in the increasing order of Partial Euclidean Distances (PEDs). Initially, the search radius $\sqrt{C}$ is set to infinity and is updated with the PED of each new candidate solution. The SE enumeration finds eligible solutions faster and is the foundation for most of SD extensions.

IV. VARIANTS OF SD ALGORITHM

The basic idea of SD algorithm is to search in a hyper sphere of radius $\sqrt{C}$ centered at the received vector y. Even though points in this hyper sphere are searched exhaustively, calculations are performed recursively, based on a search tree to enable reusing intermediate computations [10].

4.1 SE Variant of SD Algorithm

Recently, a variant of the SD algorithm appeared in both [11] and [12]. Since this version of SD algorithm was first used by Schnorr and Euchner, it is abbreviated as the SE-SDA [11]. For SE Decoder, the algorithm is based on two stages. The first stage consists in searching for the BP, which represents a first estimation, but is not necessarily, the closest point. Finding the BP gives a bound on the error. In the second stage, the BP is modified until the closest point is reached. We zigzag around each BP component in turn to build the closest point.

4.2 SD Algorithm with Increasing Radius Search (IRS)

This algorithm is initially mentioned in [13]. For a fixed search radius, there is always a probability that no candidate is found. Hence, increasing the radius is needed to achieve ML or near-ML performance while maintaining the SD algorithm's efficiency. The SD algorithm with IRS is as follows. Let $r_{p1} < r_{p2} < \ldots < r_{pn}$ be a set of sphere radii. Execute the SD algorithm with search radius r_{p1}. If a candidate is found, terminate the program; otherwise, run SD algorithm again with the next radius until r_{pn}.

4.3 SD Algorithm with Improved Increasing Radius Search (IIRS)

IRS can improve the computational efficiency of the conventional SD algorithm. However, there is an apparent waste of computations in the SD algorithm with IRS. Because for any sphere radius r_{pi}, there is always a probability that this sphere does not contain any valid lattice point. At this time, the SD algorithm increases the search radius from r_{pi} to r_{pi+1} and searches again. To reduce this loss and to lower search complexity, SD algorithm with IIRS is used.

The intuition behind the new IIRS is as follows. Whenever the SD algorithm search with radius fails, an incomplete search tree is constructed, from which promising paths can often be identified. An incomplete tree for a four dimensional (4-D) binary search is depicted in fig. 5, where the initial radius is r_{pi}. Each branch in the kth level of the tree is associated with a candidate of s. Starting from the root; each complete path corresponds to a candidate of s. From fig.5, it can be observed that paths 1 and 2 are more promising than path 3 [10].

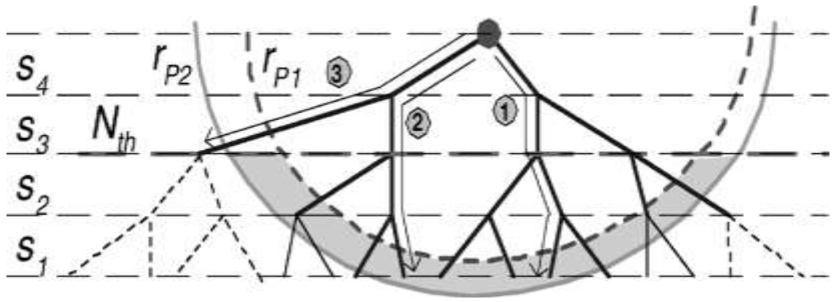

Fig.5: Search tree of the SDA [10]

4.3.1 Ordering Promising Paths

To check paths efficiently, promising paths are examined according to an ascending order of their predicted average Euclidean distances. When the SD algorithm search with radius r_{pi} fails, a path in an N-dimensional problem often consists of two segments. The first segment comprises n_1 branches, where branch metrics have been calculated. Let the sum of these branch metrics be $d^2_{n1} > r^2_{pi}$. The parameters n_1 and d^2_{n1} are generated by the SDA search with r_{pi}. The second segment comprises n_2 branches. Here, checking paths in an ascending order of their average distances maximizes the average probability to find the vector with minimum distance early.

4.3.2 Additional SD Algorithm Constraints

Ordering promising paths results in an undesirable exponentially growing memory. To reduce memory requirements, several additional constraints are assumed. First, two sphere radii $r_{pi} < r_{pi+1}$ in a single SD algorithm search are used. The radius r_{pi} plays the role of search radius as in the conventional SD algorithm, whereas r_{pi+1} upper bounds distance of promising paths when the r_{pi} search with r_{pi} fails. Second, a threshold N_{th} is used to confine the search to promising paths. This divides paths in two categories: promising or unlikely. If a path satisfies $n_2 < N_{th}$, it is a promising path and is ordered according to its average distance otherwise, it is unlikely and is ignored.

4.3.3 IIRS-A

Based on the parameters r_{pi}, r_{pi+1} and N_{th}, SD algorithm with IIRS-A called SDA-A is as follows. Before any candidate is found within r_{pi} during an SD algorithm search, the average distance for each path satisfying $n_2 < N_{th}$ and $r_{pi} < d_{n1} < r_{pi+1}$ is calculated, retaining only information about the most promising path. Whenever the SD algorithm with r_{pi} fails, SDA-A either provides information about the most promising path, or indicates that there is no such path. If the latter is true or the actual distance of the most promising path is greater than r_{pi+1}, then SDA-A fails; otherwise, it returns the candidate corresponding to the most promising path.

4.3.4 IIRS-B

To obtain near-ML performance, SD algorithm with IIRS-B called SDA-B with parameters r_{pi}, r_{pi+1} and N_{th} is as follows. Before any candidate is found within r_{pi} during an SD algorithm search, the average distance for each path satisfying $n_2 < N_{th}$ and $r_{pi} < d_{n1} < r_{pi+1}$ is calculated and complete information about the three most promising paths is kept. Whenever SD algorithm fails, SDA-B provides more information on promising paths than SDA-A. If no such path exists or no candidate within r_{pi+1} can be determined from these paths, SDA-B fails; otherwise, the first candidate found with distance < r_{pi+1} is tested. Suppose that n2 and d_{n1} are the parameters of the next promising path. If they are not available, then N_{th} and r_{pi} are used [10].

4.4 K-Best SD Algorithm

The SD algorithm can be divided into depth-first and breadth-first groups based on their search strategy. The depth-first algorithms process one candidate symbol vector at a time. The breadth-first algorithms process all the partial candidate symbol vectors on each level before moving to the next level. The K-best algorithm [14] is a breadth-first search based algorithm and keeps the K nodes which have the smallest accumulated Euclidean distances at each level. If the PED is greater than the squared sphere radius *d*, the corresponding node will not be expanded.

4.5 List Sphere Decoder (LSD) Algorithm

A LSD algorithm [15] is a variant of the SD algorithm. It provides a list of candidates L and their Euclidean distances as an output.

4.5.1 K-Best-LSD algorithm

The K-best LSD is a modification of the K-best algorithm and it outputs a list of candidate vectors and the corresponding Euclidean distances. The size N_{cand} of the output list L has an impact on the performance of the SD. With a small N_{cand}, the complexity is lower and the detection process faster, but the performance can be worse than with a full list.

4.5.2 Increasing Radius (IR)-LSD Algorithm

The IR-LSD algorithm [16] uses the metric-first search strategy. The algorithm is optimal in the sense of visited number of nodes in the tree structure. The search proceeds by calculating one branch extension at a time and stores the partial candidate to a stack memory. Then the search is always continued with the partial candidate with the lowest PED. The output of the algorithm is the candidates with lowest EDs.

V. GAP IN RESEARCH & OPEN ISSUES

The pros and cons of existing sphere decoding algorithms are shown in table 1.

Table 1: Various SD Methods

Variants of SD	Pros	Cons
SE variant of SD algorithm [17], [11], [18]	• The search phase of SE is less complex than SD. • It provides good performance	• Using QR decomposition, predecoding and initialization phases for SE are heavier than

	for a lower no. of antennas using slowly fading channels	the SD. • For lower SNR, the BP is very far from closest point, so the algorithm takes much more time to converge.
SD algorithm with IRS [1],[18],[19]	• IRS improves computational efficiency of the conventional SD algorithm. • It is effective for the medium-to-high SNR regime.	• In IRS, if for radius r_{pi}, sphere does not contain any valid lattice point then SD algorithm increases the search radius from r_{pi} to r_{pi+1}. Computations for radius r_{pi} are discarded, but they are recalculated in the search with radius r_{pi+1}. • The weakness of IRS algorithm is that it does not depend on any particular structure. In IRS, the search time for highly structured lattice is high.
SD algorithm with IIRS-A or SDA-A [18]	• It provides less decoding complexity as compared to IRS. • It provides most promising path which enables near ML performance with linear memory.	• To reduce computational complexity, IIRS-A degrades the symbol error rate (SER) performance by 0.5dB.
SD algorithm with IIRS-B or SDA-B [18]	It offers two improvements over SDA-A. • The three most promising paths are tracked by SDA-B, which enables near-ML performance with linear memory. • In SDA-B, the testing is an effective mechanism to guarantee the reliability of a candidate.	• The testing mechanism relies only on the AWGN model.
K- Best SD algorithm [14],[20]	• This algorithm is preferable in terms of hardware implementation since it has fixed complexity and memory usage.	• For this algorithm, the performance of MIMO in terms of bit error rate (BER) is degraded, especially when the number of candidate symbols

		kept at each level, is small.
K- Best LSD algorithm [21]	• The K-Best LSD algorithm guarantees a fixed throughput and complexity.	• The K-Best LSD requires a larger list size compared to the IR-LSD to obtain as accurate approximation and performance.

From study of existing algorithms and their pros and cons, it can be concluded that these algorithms show the following major weaknesses:

- The sphere decoder performance is very sensitive for the most current proposals in order to choose the search radius parameter. The successful termination of the algorithm which provides the result as an optimal solution is highly dependent on the search radius.
- Secondly, at high spectral efficiencies which are required to support higher communication rates, SNR is low and the complexity coefficient can become very large.

The issues for future research in field of sphere decoder are given below:

- To improve the performance of MIMO-OFDM using existing / modified detection.
- To improve the BER performance and to reduce computational complexity for existing sphere decoding algorithms.

VI. CONCLUSION

Different SD algorithms along with their modified forms have been studied. The existing search strategies are described in a unified framework and their pros and cons have been discussed.

REFERENCES

[1] P.W Wolniansky, G. J. Foschini, G. D. Golden and R. A. Valenzuela, V-BLAST: an architecture for realizing very high date rates over the rich-scattering wireless channel, in *International Symposium on Signals, Systems and Electronics*, pp. 295-300, 1998.

[2] H. Liao, A zero forcing and sphere decoding joint detector for multiple input multiple output LAS-CDMA system, *IEEE WRI International Conference on Communications and Mobile Computing*, pp. 246-249, 2009.

[3] E.G. Larsson, MIMO detection methods: How they work, *IEEE Signal Processing Magazine,* May 2009.

[4] K. Su, I. Berenguer, I. J. Wessell and X. Wang, Efficient maximum-likelihood decoding of spherical lattice codes, *IEEE Transactions On Communications, vol.57, no. 8,* pp. 2290-2300, Aug. 2009.

[5] R. P. Yadav, G. Mathur, S. Mrinalee and H. P. Garg, Improved radius selection in sphere decoder for MIMO system, *International Conference on Computing for Sustainable Global Development,* pp. 183-186, March 2014.

[6] MD.S.Alam, On sphere detection for OFDM based MIMO systems, *Blekinge Institute of Technology*, pp. 33-37, Sept. 2010.

[7] U.Fincke and M.Pohst, Improved methods for calculating vectors of short length in lattice including a complexity analysis, *AMS Mathematics of Computation, Vol. 44, No. 170*, pp. 463-471, April 1985.

[8] C. Schnorr and M. Euchner, Lattice basis reduction: Improved practical algorithms and solving subset sum problems, *Springer, Mathematical Programming, Vol. 66, Issue 1-3*, pp. 181–199, Aug. 1994.

[9] N. K. Noordin, B. M. Ali, S. S. Jamuar and M. Ismail, A simplified sphere decoding algorithm for MIMO transmission system, *University Putra, Malaysia*.

[10] W. Zhao and G. Giannakis, Sphere decoding algorithms with improved radius search, *IEEE Trans. Commun.*, *vol. 53*, pp. 1104–1109, July 2005.

[11] E. Argell, E. Eriksson, A. Vardy and K. Zeger, Closest point search in lattices, *IEEE Trans. Inform. Theory*, *vol. 48*, pp. 2201–2214, Aug. 2002.

[12] G. Rekaya and J.-C. Belfiore, Complexity of ML lattice decoders for the decoding of linear full rate space-time codes, *IEEE Trans. Inform. Theory*, pp. 206, July 2003.

[13] E. Viterbo and J. Boutros, A universal lattice code decoder for fading channel, *IEEE Trans. on Inform Theory, vol. 45, no. 5*, pp. 1639-1642, July 1999.

[14] K. Wong, C. Tsui, R.-K. Cheng and W. Mow, A VLSI architecture of a K-best lattice decoding algorithm for MIMO channels, in *Proc. IEEE Int. Symp. Circuits and Systems*, *Helsinki*, *Finland*, *vol. 3*, pp. 273–276, June 2002.

[15] B. Hochwald and S. ten Brink, Achieving near-capacity on a multiple-antenna channel, *IEEE Transactions on Communications*, *vol. 51, no. 3*, pp. 389–399, March 2003.

[16] M. Myllyla, J. Cavallaro and M. Juntti, A list sphere detector based on Dijkstra's algorithm for MIMO-OFDM systems, in *Proc. IEEE Int.Symp. Pers., Indoor, Mobile Radio Commun. (PIMRC), Athens, Greece,* Sept. 12 - 19, 2007, pp. 1-5.

[17] A. M. Chan and I. Lee, A new reduced-complexity sphere decoder for multiple antenna system, *IEEE Conference on Communications*, *ICC2002*, pp.460-464.

[18] M.O. Damen, H. E. Gamal and G. Caire, On maximum-likelihood detection and the search for the closest lattice point, *IEEE Transactions on Information Theory, Vol. 49, No. 10*, pp. 2389-2402, Oct. 2003.

[19] B. Hassibi and H. Vikalo, On the sphere-decoding algorithm I. expected complexity, *IEEE Transactions on Signal Processing, Vol. 53, No. 8,* pp.2806-2818, Aug. 2005.

[20] B. Shim and I. Kang, On radius control of tree-pruned sphere decoding, in *Proceedings of the IEEE International conference on Communications*, pp. 2469-2472, 2009.

[21] M. Myllyla, M. Juntti and J. R. Cavallaro, Implementation aspects of list sphere decoder algorithms for MIMO-OFDM systems, *Signal Processing 90*, April 2010, pp. 2863–2876.