STUDY OF EQUALIZATION TECHNIQUES FOR MIMO SYSTEM

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ABSTRACT
The wireless communication systems offer high Data rate, Wide coverage and improved reliability. To achieve these in wireless communication the MIMO systems are used that can efficiently increases the data transmission rate and the system coverage by considering multiple numbers of transmitter antennas and receiver antennas and spatial dimensions are used. When the data is transmitted at high rates, due to this Inter-symbol interference (ISI) occurs. To eliminate this effect different equalization techniques are used. Equalization techniques such as Zero forcing equalizer (ZFE), minimum mean square error (MMSE) and maximum likelihood (ML) detection algorithms can be employed. This study will analyse the performance of equalization techniques by considering multiple transmit and multiple receive antenna over a flat fading Rayleigh channel and gives a bit error rate analysis of the same. MIMO system transmission with Binary Phase Shift Keying Modulation (BPSK) in Rayleigh fading channel is considered.

Keywords: Bit error rate (BER), Multiple input multiple output (MIMO), Maximum likelihood (ML), Minimum mean-squared error (MMSE), Zero Forcing (ZF).

I. INTRODUCTION
High data rate in wireless system causes intersymbol Interferences [1]. MIMO techniques extend the promise of high spectral efficiency and hardness to fading. Key to their success is the MIMO equalizers used at the receiver [2], which will recover the symbols that are transmitted simultaneously from multiple antennas. Equalizers are categorized into linear and non-linear equalizers. Linear equalizers are ZF which implements matrix (Pseudo)-inverse (ignores noise enhancement problems) and MMSE optimizes the noise and offers a compromise between residual interference between input signals and noise enhancement. Non-linear receivers are ML. ML is exhaustive optimum detection equalizer uses complexity exponential in QAM. This paper coverage ranges from simple linear equalizers based on the zero-forcing and MMSE criteria to the optimal maximum-likelihood are described.

The focus of this paper is on the development of such novel practical, low complexity equalization techniques and understanding their potential and limitations when used in wireless communication systems characterized by very high data rates, high mobility and the presence of multiple antennas

II. EQUALIZATION

When the signal passes through the channel, distortion is introduced in terms of amplitude and delay, which results with Inter Symbol Interference (ISI). ISI distorts the transmitted data, causing bit errors at the receiver. ISI
has been recognized as the major drawback in high speed data transmission over wireless channels. Hence, Equalizers used to combat ISI [3]. An equalizer is implemented at the baseband [5] or at IF in a receiver. And the basic receiving technique of any communication lies with noise signal performance. Hence to suppress the noise component present in communication [4], we should go for equalization technique. Since the baseband complex envelope expression can be used to represent band pass waveforms, the channel response, demodulated signal and adaptive equalizer algorithms are usually simulated and implemented at the baseband [3].

III. MIMO SYSTEM

For 2×2 MIMO system
The received symbols after transmission of symbols in first time slots can be given as the received [4] symbol on first receive antenna is,

\[ y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \, h_{1,2}] [x_1 \, \, x_2] + n_1 \] (1)

The received signal on the second receive antenna is,

\[ y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \, h_{2,2}] [x_1 \, \, x_2] + n_2 \] (2)

\[
H = \begin{bmatrix}
    h_{1,1} & h_{1,2} \\
    h_{2,1} & h_{2,2}
\end{bmatrix}
\] (3)

Where \( y_1, y_2 \) are the received symbols on the first and second antenna respectively, \( h_{1,1} \) is the channel from 1st transmit antenna to 1st receive antenna, \( h_{1,2} \) is the channel from 2nd transmit antenna to 1st receive antenna, \( h_{2,1} \) is the channel from 1st transmit antenna to 2nd receive antenna, \( h_{2,2} \) is the channel from 2nd transmit antenna to 2nd receive antenna, \( x_1, x_2 \) are the transmitted symbols and \( n_{1,2} \) is the noise on 1st, 2nd receive antennas.

Above equation can be written as

\[ y = Hx + n \] (4)

Here \( Y \) is the received symbol
\( H \) is a Rayleigh fading channel matrix
\( X \) the transmitted symbol matrix
\( N \) is the noise matrix

IV. LINEAR EQUALIZERS

There are two types of linear equalizers: Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) which shall be discussed shortly. The former equalizer cancels all ISI, but can lead to considerable noise enhancement. The latter technique minimizes the expected mean squared error between the transmitted symbol and the symbol detected at the equalizer output, thereby providing a better balance between ISI mitigation and noise enhancement [12]
4.1 Zero Forcing (ZF) Equalizer

The ZF Equalizer is a linear equalization algorithm used in communication systems proposed by Robert Lucky. It works by inverting the frequency response of the channel. In order to restore the signal before the channel, the equalizers apply the inverse of the channel to the received signal. The name Zero Forcing corresponds to bringing down the Inter Symbol Interference (ISI) to zero. But while doing so there may be an amplification of the noise term present [1].

The ZF equalizer is simple to implement but suffers from the problem of noise amplification hence not suitable for channels with high noise characteristics [1]

**Mathematical Modeling of ZF equalization Matrix**

In matrix notation:

\[
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix} =
\begin{bmatrix}
    h_{1,1} & h_{1,2} \\
    h_{2,1} & h_{2,2}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    n_1
\end{bmatrix} +
\begin{bmatrix}
    n_2
\end{bmatrix}
\]  

(5)

Equivalently,

\[
y = Hx + n
\]

To solve for \(x\), there is the need to find a matrix \(W\) which satisfies \(WH = 1\). The Zero Forcing (ZF) detector for meeting this constraint is given by,

\[
W = (H^H H)^{-1} H^H
\]

(6)

Where: \(W\) - Equalization Matrix, and \(H\) - Channel Matrix. This matrix is known as the Pseudo inverse for a general\( m \times n\) matrix.

4.2 Minimum Mean Square Error (MMSE) Equalizer.

The MMSE equalizer is based on the Minimum Mean Square Error algorithm which tries to obtain the Mean Square Error (MSE), which is a common measure of estimator quality. The main feature of MMSE equalizer is that it does not usually eliminate ISI completely but, tries to provide a trade off between ISI mitigation and noise enhancement by minimizing the total power of the noise and ISI components in the output.

Let \(x\) denote the original transmitted symbol, and \(x^\wedge\) be the estimate of the transmitted symbol at the output of the equalizer. The MMSE equalizer tries to find a coefficient which minimizes the MSE between the original information symbol and the output of the equalizer as

**Mathematical Modelling of MMSE Equalization Matrix**

In matrix notation:

\[
\begin{bmatrix}
    y_1 \\
    y_2
\end{bmatrix} =
\begin{bmatrix}
    h_{1,1} & h_{1,2} \\
    h_{2,1} & h_{2,2}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} +
\begin{bmatrix}
    n_1 \\
    n_2
\end{bmatrix} 
\]

(7)

Equivalently,

\[
y = Hx + n
\]

To solve for \(x\), there is the need to find a matrix which satisfies . The Minimum Mean Square Error (MMSE) detected for meeting this is constraint by

\[
W = (H^H H + N_0 I)^{-1} H^H
\]

(8)

Where -Equalization Matrix and \(H\) - Channel Matrix. \(W\)
In fact, when the noise term is zero, the MMSE equalizer reduces to the Zero Forcing equalizer. This model can be extended to m x n antenna configuration

V. NON-LINEAR EQUALIZERS

Non-Linear equalizers are implemented using a lattice structure. The lattice filter employed in this type of equalizer has a more complex recursive structure than the transversal filter used by the linear equalizers. In exchange for this increased complexity relative to transversal structures, lattice structures often have better numerical stability and convergence properties and greater flexibility in changing their length.

5.1 Maximum Likelihood (ML) Equalizer

The problem with most equalizers especially linear equalizers is the fact that they are not optimal in terms of minimizing the average symbol error probability. Because of the fact that the effect of a symbol is spread to other symbols, it is intuitive that the optimal receiver should observe not only the segment of received signal concerning the desired symbol, but the whole received signal instead. Thus, the Maximum Likelihood equalizer provides the optimal solution to equalization problems by minimizing the probability of error over the entire sequence.

The ML[6] equalizer chooses the transmitted input sequence that maximizes the log-likelihood of the received signal sequence at the receiver when the channel information is been known at receiver side. The algorithm was first proposed by Forney and later implemented with Viterbi algorithm to reduce computational complexity.

5.2 Mathematical Modeling of (ML) Equalizer

The ML equalization approach determines the estimate of the transmitted signal vector x as [14]

\[ K = |y - Hx|^2 \]  \hspace{1cm} (9)

Expanding,

\[ K = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} - \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]  \hspace{1cm} (10)

Using BPSK modulation technique, the possible values of is +1 or -1. Similarly also take values +1 or -1. In finding the Maximum Likelihood solution, there is the need to find the minimum from the all four possible combinations of and x1 and x2.

VI. MIMO SYSTEM MODEL

![MIMO system Model](image-url)
VII. CONCLUSION

In this paper, we study the performance of three types of equalizers namely, ZF, MMSE and ML equalizers for MIMO wireless receiver. Study analysis shows that the maximum likelihood detection equalizer plays a very vital role to improve the performance of the MIMO system than ZFE and MMSE equalizers.

REFERENCES