



THEORETICAL ANALYSIS AND MATHEMATICAL MODELLING OF PARABOLIC TROUGH CONCENTRATING PHOTOVOLTAIC THERMAL SYSTEM

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ABSTRACT

The electricity and heat produced simultaneously in a photovoltaic thermal (PVT) system from solar energy is about 60-70% efficient. The traditional photovoltaic (PV) system conversion of electricity from solar energy is only about 6-15% efficient, whereas 85% of the incoming solar energy is either reflected or absorbed as heat energy, which are cooled by water or air coolant to utilize the all incoming solar energy on system. The main objectives in this project work is combining two systems; Parabolic Trough as a concentrator and channel PV/T collector as a receiver. The theoretical analysis and mathematical modelling of parabolic trough concentrating photovoltaic thermal system was done and evaluated through water flow with concentrating PVT.

Keywords: *Theoretical and Mathematical Modelling, Solar Pvt*

I. INTRODUCTION

The most important factor consider in electrical efficiency of photovoltaic (PV) cell are related to the band gap. Photon emitted from sun with energy below the band gap energy cannot be absorbed by PV and is transmitted. Photon with energy greater than the band gap energy is absorbed and converted into electricity. But that excess energy is lost to heat in the PV. Due to this heat, some losses are occurring in PV such as ohmic losses.

Also PV cell absorb up to 80% of the solar irradiation. However, only 5–20% of the incident energy is converted into electricity. The remaining energy is converted into heat. So here we needed to use remaining 60% to 75% of incident energy to any heat cycle for high efficiency of PV cell. On sunny days PV laminates can reach temperatures as high as 35 °C above ambient temperature. In PV/T system, this heat is extracted from the PV panel and made available for use in a building, e.g., for tap water heating and space heating. With an optimal design, PVT systems can supply buildings with 100% renewable electricity and heat in a more cost-effective manner than separate PV and solar thermal systems (Helden, 2004).

1.1 Principle of Electricity Generation by Photovoltaic Cells

A photovoltaic cell comprises P-type and N-type semiconductors with different electrical properties, joined together. The joint between these two semiconductors is called the "P-N junction." Sunlight striking the

photovoltaic cell is absorbed by the cell. The energy of the absorbed light generates particles with positive or negative charge (holes and electrons), which move about or shift freely in all directions within the cell.

The electrons (-) tend to collect in the N-type semiconductor, and the holes (+) in the P-type semiconductor. Therefore, when an external load, such as an electric bulb or an electric motor, is connected between the front and back electrodes, electricity flows in the cell.

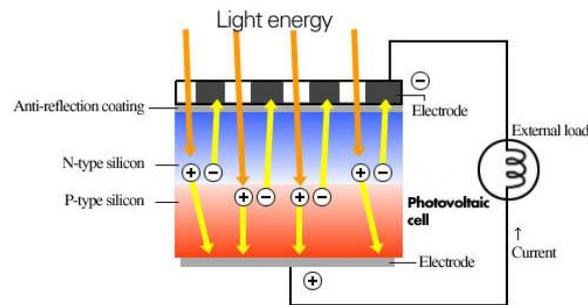


Fig. 1 Photovoltaic cell generates electricity when irradiated by sunlight.

1.2 Thermophotovoltaics

Thermophotovoltaics cell uses different technology to produce electricity. Thermo- means heat, these cells converts heat into electricity; rest of it works as same as photovoltaic cells which converts light into electricity.

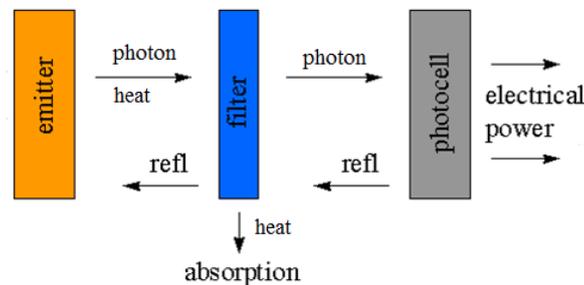


Fig. 2 Block diagram of thermophotovoltaics.

The only difference between thermo-photovoltaic and photovoltaic is that thermo photovoltaic cells uses semiconductor which are designed for long wavelength, invisible light like infrared rays released by hot objects. This way of generating electricity is very neat and clean and also simpler to what we experience in power generation using generators, steam turbines etc.

1.3 Parabolic Trough Collector

A parabolic trough is a type of solar thermal energy collector. It is constructed as a long parabolic mirror (usually coated silver or polished aluminum) with a Dewar tube running its length at the focal

point. Sunlight is reflected by the mirror and concentrated on the Dewar tube. The trough is usually aligned on a north-south axis, and rotated to track the sun as it moves across the sky each day.

The first practical experience with PTCs goes back to 1870, when a successful engineer, John Ericsson, a Swedish immigrant to the United States, designed and built a 3.25-m²-aperture collector which drove a small 373-W engine. Steam was produced directly inside the solar collector (today called Direct Steam Generation or DSG). From 1872 to 1875, he built seven similar systems, but with air as the working fluid

To deliver high temperature with good efficiency a high performance solar collector is required. System with light structure and low-cost technology for process heat application up 400°C could be obtained with parabolic trough collector (PTC). PTC can effectively produce heat at temperature between 50°C and 400°C. Parabolic trough is made by bending a sheet of reflective material to parabolic shape. When the parabola is pointed towards the sun, parallel rays incident on the reflector and reflected on to the receiver. The concentrated radiation reaching the receiver tube heats the fluid that circulates through it, thus transforming the solar radiation into useful heat. It is sufficient to use a single axis tracking of the sun. The collector can be oriented in an east west direction, tracking the sun from north to south, or in a north-south direction, tracking the sun from east to west.

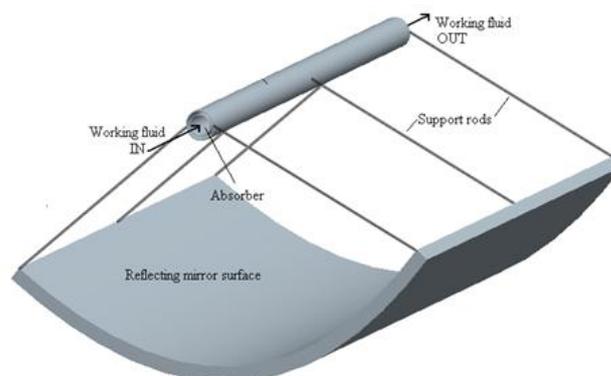


Fig. 3 Parabolic Trough Collector

II. LITREURE SURVEY

A concentrated photovoltaic thermal (CPVT) system is a combination of photovoltaic (PV), solar thermal systems and solar concentrators which produce high grade electricity and heat from one integrated system. In other words, Those PV panels, solar thermal systems and solar concentrators operating side by side are therefore, the space required for separated system is high compare to the combined system (Clarke, 1996). These are alternative ways to improve the cost effective of combined system (Helden, 2004). Among many others, there can be selections among trough concentrating, Fresnel linear concentrator, parabolic concentrator, air, water or evaporative collectors, monocrystalline / polycrystalline / amorphous silicon (c-Si/pc-Si/a-Si) or thin-film solar cells, flat-plate or concentrator types, glazed or unglazed panels, natural or forced fluid flow. Accordingly, available installations are ranging from PVT air and/or water pre-heating system to hot water



supply through PV integrated heat pump, and to actively-cooled PV concentrator through the use of economical reflectors.

Theoretical and experimental studies of PV/T were documented as early as in mid 1970s (Hendrie, 1982). In the PV/T system heat removing from the PV cell by means of heat absorbing medium such as water, flowing in pipes.

The heat removed from cells by water is used as the hot water for domestic purpose. By this the overall efficiency is higher and lower packaging cost due to its compact design (Braunstein, 1986). During some severe cold days in winter, anti-freeze liquid can be used. But it cannot be used in summer because the heat transfer rate in liquid is less, so the performance of the anti-freeze liquid is slightly less in summer (Norton, 1991).

The detailed physical model for the flat-plate PVT/water (PVT/w) collector system performance is evaluated. The fin-width to tube-diameter ratio was investigated and the total efficiency was found in the range of 60-80% (Bergene, 1995).

2.1 Glass

The single-covered PVT design was found better than uncovered PVT design (of which the thermal efficiency is unfavorable) or the double covered PVT design (of which the cell efficiency is unfavorable). The exergy analysis performed, indicated that the exergy output density of the uncovered design is slightly higher than the single-covered design, taking the fact that the thermal energy contains much unavailable energy (Fujisawa, 1991). For some low-temperature water-heating system, like swimming pool applications, the low-cost unglazed PVT/w system is recommended.

Working with a steady state PVT/air (PVT/a) modal, point out that the increased transmission losses – beyond the critical point the single-glass cover collect more heat than the double-glass. The exergy analysis performed, indicated that the exergy output density of the uncovered design is slightly higher than the single-covered design, taking the fact that the thermal energy contains much unavailable energy (Garg, 1997). Based on the weather of New Delhi, their transient simulation analysis found that in terms of overall energy performance, the double-glass configuration is better than the single-glass option for conventional PVT/a collector (Garg, 1998).

2.2 PV cell

The lower cost of amorphous silicon solar modules as compared to other silicon based techniques. Furthermore, it has been shown that the temperature coefficient of amorphous silicon solar cells (-0.1%K) is much smaller than that of crystalline silicon solar cells (-0.4%K). This allows to operate a-Si:H modules at temperature up to 1000c without substantial power losses (Fujisawa, 1997; Ibrahim, 2009). The efficiency of amorphous silicon solar and crystalline silicon solar cell is 5% and 12% respectively (Daghigh, 2009).



2.3 Absorber

Simulations were performed to determine the best absorber design that gives the highest efficiency (total efficiency) of PVT (see Fig. 2.4). In these simulations, the system is analyzed with various parameters, such as solar radiation, ambient temperature, and flow rate conditions. It is assumed that the collector is represented as a flat plate thermal collector with single glazing sheet. Based on these simulations, spiral flow design proved to be the best design with the highest thermal efficiency of 50.12% and corresponding cell efficiency of 11.98% (Ibrahim, 2009). The overall efficiency of PV/T is improved, when PV cells were laminated upon the solar thermal metallic coating panels (Hollick, 1998).

The PV/T air system consists of monocrystalline silicon cells pasted to an absorber plate with fins attached at the other side of the absorber surface. Air as heat removing fluid is made to flow through an upper channel and then under the absorber plate. Only a small part of the absorbed solar radiation is converted to electricity, while the rest increases the temperature of the PV cells. Improvements to the total efficiency of the PV/T air system can be achieved by the use of a double-pass collector system and fins (Othmana, 2007). An experimental investigation of a solar air heater with photovoltaic cell located at the absorber with compound parabolic collector (CPC) and fins have been developed and tested (Alfegi, 2008).

2.4 Working fluid

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(Ming Li, February 2011) analysed the electrical and thermal performance of a 2 m² Trough Concentrating Photovoltaic/Thermal (TCPV/T) system with an energy flux ratio 10.27 using single crystalline silicon solar cell array, a polycrystalline silicon cell array, a Super cell array and a GaAs cell array respectively. The experimental results show that the electrical performance of the system with the GaAs cell array is better than that of crystal silicon solar cell arrays. The superior output performance of the GaAs cell array mainly benefits from its lower series resistance. But the thermal performances of the system using the single crystal silicon solar



cell array and the polycrystalline silicon solar cell array are better. He used another 10 m² TCPV/T system with an energy flux ratio of 20 using the GaAs cell array and a concentrating silicon cell array.

The experimental results indicated that the photoelectric efficiency of the GaAs cell array is 23.83%, and the instantaneous electrical efficiency and thermal efficiency of the system are 9.88% and 49.84% respectively. While the instantaneous electrical efficiency and thermal efficiency of the system using the low-cost concentrating silicon cell array are 7.51% and 42.4% respectively.

(Xiang et al 2009) investigated the performance on trough concentrating solar photovoltaic/thermal system based on super cells and the mathematic model of a trough concentrating solar PV/T system, next he established and verified with experimental data. With internal and external parameters, schematically he analyzed the performance of PV/T system and the trough concentrating solar PV/T system based on super cells. He included the internal parameters, optical efficiency of mirror, width of focusing line, thermal conductivity of heat sinking tape and thermal absorption of the lighting plate, while the external parameters were included with wind speed and the solar direct radiation. Subsequently he undertake large number of simulations to test the thermal, electricity, total and exergy efficiency of PV/T system for variation of the design parameters. The result indicated that the optical efficiency of mirror has more effect on the performance of system than others, the total and exergy efficiency increase 0.9 times and 0.5 times respectively with the optical efficiency of mirror increasing from 0.5 to 0.95.

(Xu et al 2008) Experimentally studied on solar cell arrays of trough concentrating system and conducted a series experiment with single crystalline silicon solar cell array, polysilicon solar cell array, super cell array and GaAs cell array using trough concentrating system. His experimental results show that the I-V curve of the crystalline silicon, polysilicon and super cell arrays are beeline in the concentrating solar irradiance, which reduce the efficiency and decrease the output power. However the I-V curve of GaAs cell array is very good in the concentrating solar irradiance, the efficiency of GaAs cell array increases from 23.66% to 26.50%, and the output of GaAs magnifies 12 times. As a result he found that, it is considerable to improve the efficiency if the GaAs cell has been adopted in the concentrating PV system, with temperature coefficients of P_m , FF and η of GaAs cell array of -0.12W/K, -0.10%/K, and -0.21%/K. In order to keep better efficiency, he suggested for the forced cooling system, at the same time he recovered the quantity of heat for using.

(Garg et al 1999) Analysed the performance of a hybrid photovoltaic/thermal (PV/T) collector with integrated compound parabolic concentrator (CPC) troughs and theoretically analyzed the modeling of thermal and electrical processes of a hybrid PV/T air heating collector coupled with a CPC. In this design, several CPC troughs are combined in a single PV/T collector panel. The absorber of the hybrid PV/T collector under investigation consists of an array of solar cells for generation of electricity, while collector fluid circulating past the absorber provides useful thermal energy as in a conventional flat plate collector. In the analysis, he assumed that solar cell efficiency can be represented by a linear decreasing function of its temperature. Based on the



developed analysis, he presented and discussed that both thermal and electrical performance of the system are the function of system design parameters

III. PROBLEM IDENTIFICATION

There are two main problems associated with using the solar radiation concentrators in TCPV/T systems, as discussed below:

- (i) In TCPV/T systems, the efficiency of the system is based on the proper focusing of solar radiation over the surface of the receiver (PV cell). Whereas in the existing work PV cell are arranged in horizontal plane and solar radiation is focused on to it in inclined direction, by this arrangement the PV cell does not receives solar radiation in perpendicular direction which may causes some reflection losses.
- (ii) Concentration of solar radiation on the surface of PV cell increases the cell's temperature. Cooling of the cell by natural convection is generally effective under one sun condition in most locations. However, under a concentrated radiation the conventional natural cooling methods fail to maintain the cell's temperature at a reasonable level, as a result the efficiency substantially drops. Attempts have been made to actively cool cells, for example by running water. This then will require the availability of water and the added cost of complexity of a pump.

IV. A NEW DESIGN FOR CPVT

Table 1. New design of CPVT through the comparative study on literature review

Parameter	Types	Description
Glass	glass cover	Beyond the critical point the single-glass cover collect more heat than the double-glass. Uncovered design (of which the thermal efficiency is unfavorable) The double covered design (of which the cell efficiency is unfavorable). (Skoplaki, 2009)
PV cell	Si	0.4% per $^{\circ}\text{C}$ rise for Si cells. Efficiency is 11.98 (Ibrahim A, 2009) 0.1% per $^{\circ}\text{C}$ rise for a-Si:H cell (Fujisawa, 1997). Efficiency is 5 (Daghigh, 2009)
Design of PVT	Channel below-transparent-PV design	The channel-below-transparent-PV design gives the best efficiency. The thermal efficiency is 0.63 and electrical efficiency

		is 0.09 (Zondag, 2003)
Working fluid	Water	Compare to air, water is good thermal absorber (Prakash, 1994)
concentrator	Reflective mirrors	Improved the overall efficiency of PV/T, when PV cells were focused on reflective mirrors. (Ming Li, Mar 2011)
Collector	Parabolic Trough	Parabolic trough can provide high temperature heat from 200 to 250°C for SEGS (Solar Electric Generating Systems) plants and CPS plants and many other applications (A.Ferna´ ndez-Garci´ a. 2010)

V. THEORETICAL ANALYSIS AND MATHEMATICAL MODELLING

The theoretical and simulation analysis for determination of the concentrated PVT system's electrical and thermal efficiency, is calculated or simulated before installing in the site for avoid PV module overheating by the concentrated solar intensity that is converted into heat instead of electricity. The concentration ratio of parabolic trough concentrated PVT system is calculated by:

$$CR = A_{\alpha-sd} / A_{pvt} \quad (5.1)$$

where $A_{\alpha-sd}$ is the aperture area of trough and $A_{\alpha-pvt}$ the aperture area of the PVT model. The electrical efficiency η_{el} depends mainly on the incoming solar intensity I and the PV module temperature (T_{pv}) and is calculated by:

$$\eta_{el} = I_m V_m / I A_{\alpha-pvt} \quad (5.2)$$

where I_m and V_m are the current and the voltage of the PV module operating at maximum power. The formula that can be used for calculation of the PV module temperature is a function of the ambient temperature T_a and the incoming solar intensity I and is given by (Lasnier and Ang, 1990) as:

$$T_{pv} = 30 + 0.0175(I - 300) + 1.14(T_a - 25) \quad (5.3)$$

The traditional linear expression for the PV electrical efficiency is given by (Evans and Florschuetz, 1977) as

$$\eta_{el} = \eta_{T_{ref}} [1 - \beta_{ref}(T_{pv} - T_{ref})] \quad (5.4)$$

in which $\eta_{T_{ref}}$ is the module's electrical efficiency at the reference temperature, T_{ref} , and at solar intensity of 1000 W/m² (Evans, 1981). The present value of reference efficiency is 12% at temperature of 25. The

temperature coefficient (β_{ref}) is the main material properties, having values of about 0.004 K^{-1} respectively, for crystalline silicon modules (Notton et al., 2005). T_{pv} is the average temperature of cells. The Fig 4 shows the electrical efficiency characteristic in graph with concentrated and non – concentrated solar intensity by using equation (5.4). According to the graph, the electrical efficiency is decreased when solar intensity is increased. This is because of intensity convert into heat in the PV cell instead of convert into electricity which is shown in Fig. 6.1. The ohmic loss occurs in the electricity generation in PV cell, this can be avoided by using coolant flow below or above the PV cell. Coolant (water or air) removes heat from the PV cell and efficiency is increased. We designed a PVT system with coolant flow of water below the PV cell which is transparent to PV cell, according to design of Zondag et al (2003).

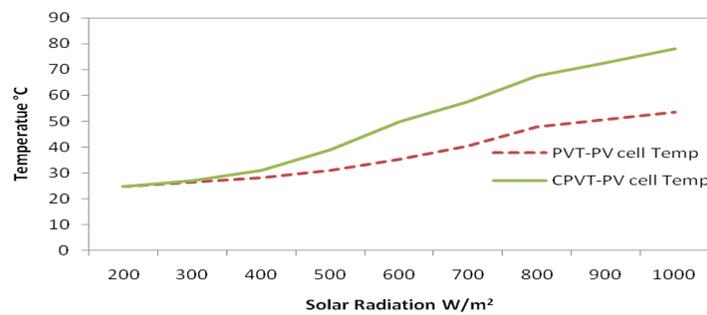


Fig. 4. Theoretical PV cell Temperature for PVT and CPVT.

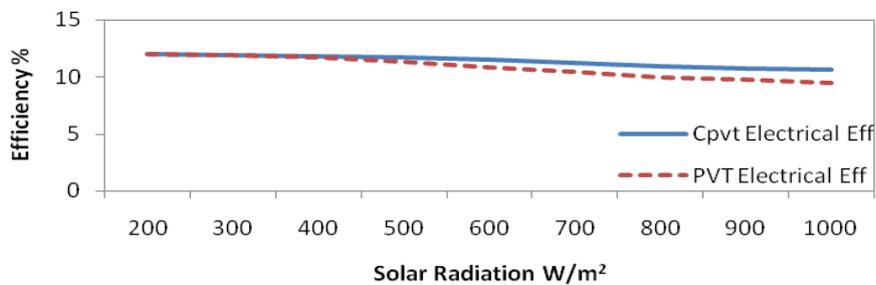


Fig. 5. Theoretical electrical efficiency of PVT and CPVT.

The thermal efficiency of designed PVT system is calculated in the steady state condition as given by:

$$\eta_{th} = \dot{m}C_{p-w}(T_o - T_i) / IA_{a-pvt} \quad (5.5)$$

where \dot{m} is the fluid mass flow rate, C_{p-w} the fluid specific heat, T_i and T_o the input and output fluid temperatures, respectively and η_{th} is the thermal efficiency of PVT. For the analysis, the outlet temperature of coolant flow is unable get value without fabrication. Instead of using the equation (5.5), we can calculate the thermal efficiency as a function of the ratio $\Delta T/I$, where $\Delta T = T_i - T_a$, with T_a being the ambient

temperature. According to the book of Garg and Prakash (1992, p 55), the instantaneous thermal efficiency of flat plate collector is given as:

$$\eta_{i-t\dot{h}} = F_R (\tau\alpha)_{pv} - F_R U_{l-pvt} (T_i - T_a)/I \quad (5.6)$$

where, F_R is the heat-removal factor which is connected with the efficiency factor (F_f) using the following equation:

$$F_R = IC_p/U_L \left[1 - \exp\left(-\frac{U_L F_f}{IC_p}\right) \right] \quad (5.7)$$

where, (F_f) varies with different types of working mediums (e.g., water or air):

for coolant flowing in circular pipes

$$F_f = \frac{1/U_L}{W[1/U_L[D_o+(W-D_o)F]+1/C_b+1/\pi D_i h_{wm}]} \quad (5.8)$$

for coolant flowing in duct

$$F_f = \frac{1}{1+[U_L/(h_{wm} A_{HT}/A_{a-pvt} + 1/(1/h_r+1/h_{wm}))]} \quad (5.9)$$

where, W is the distance between tubes; D_o and D_i are the outside and inside diameter of flow tubes; C_b is conductance of the bond between the fin and tube; h_{wm} is the heat transfer coefficient of working medium; A_{HT}/A_{a-pvt} is the ratio of heat transfer area to collector aperture area; h_r is the equivalent radiation coefficient. In our design the water is flowing in the rectangular area under the PV glass which as like single pass PVT air heater. Equation (5.6) gives the thermal efficiency of flat plate collector, in which absorber is replaced as PV cell laminated on absorber plate. Instead using the absorber plate mean temperature for calculation, in our system PV cell temperature is considered, because PV cell and absorber plate act as one single structure and which is calculated using the equation (5.3). The overall heat loss coefficient U_L of the system is made from three components, the bottom loss coefficient U_b , the edge loss coefficient U_e and the top loss coefficient U_t is calculated by equation given by Garg and Prakash (1992, P 59-65) as:

$$U_L = U_b + U_e + U_t \quad (5.10)$$

$$U_b = \frac{K_i}{l_i} \quad (5.11)$$

where, K_i is the thermal conductivity of insulation; l_i is the thickness of the insulation.

$$U_e = \frac{l_g K_i (l_1 - l_2)}{l_1 l_2 l_i} \quad (5.12)$$



where, l_1 is the length of the collector; l_2 is the width of the collector; l_3 is the thickness of the collector.

$$U_t = \left[\frac{N}{\left(\frac{C}{T_m}\right)(T_m - T_a)^{0.2522}} + \frac{1}{h_w} \right]^{-1} + \left[\frac{\sigma(T_m^2 + T_a^2)(T_m + T_a)}{\frac{1}{\varepsilon_p + 0.0425N(1 - \varepsilon_p)} + \frac{2N + f - 1}{\varepsilon_g} - N} \right] \quad (5.13)$$

$$f = \left[\frac{9}{h_w} - \frac{30}{h_w^2} \right] \left[\frac{T_a}{316.9} \right] (1 + 0.091N) \quad (5.14)$$

$$h_w = 2.8 + 3V_{wind} \quad (5.15)$$

$$C = 520(1 - 0.000051\varphi^2) \text{ for } 0^\circ < \varphi < 70^\circ \quad (5.16)$$

where, ε_g is the emittance of glass; ε_p is the emittance of absorber plate; T_m is the mean plate temperature (k); h_w is the wind heat transfer coefficient (W/m²K); N is the number of glass cover; φ is the collector tilt from horizontal; V_{wind} is the velocity of wind.

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