International Journal of Electrical and Electronics Engineers
Volume 15, Issue No. 02, July-Dec 2023
ISSN (O) 2321-2055
ISSN (P) 2321 -2045

The Revolutionizing Image Representation and Compression with Advanced Quantum DCTEFRQI Approach

Mohd Faisal[1], Dr. Pushpneel Verma[2]

1 Research Scholar, Dept. of Computer Science & Engg., College of Engineering & Technology Bhagwant University Ajmer, Rajasthan, India, M_faisalcse@rediffmail.com

2. Dept. of Computer science & Engg , College of Engineering & Technology, Bhagwant Group of Istitutions, Ajmer, Rajasthan, India. pushpneelverma@gmail.com

ABSTRACT

Quantum image processing has gained significant attention in recent years due to the ability to process large image data in quantum Hilbert space, which has infinite dimensions and allows for faster image processing. However, encoding and compressing images in the quantum domain remains a challenging issue. In this study, we propose a Direct Cosine Transform Efficient Flexible Representation of Quantum Image (DCTEFRQI) algorithm to efficiently represent and compress gray images, which saves computational time and minimizes complexity. Our objective is to represent and compress various gray image sizes in a quantum computer using a combination of Discrete Cosine Transform (DCT) and Efficient Flexible Representation of Quantum Image (EFRQI) approaches. We use the Quirk simulation tool to design the corresponding quantum image circuit, and a total of 16 qubits are used to represent the gray scale image, with 8 used to map the coefficient values and the remaining 8 used to generate the corresponding coefficient position. Theoretical analysis and experimental results show that the proposed DCTEFRQI scheme provides better representation and compression compared to DCT-GQIR, DWT-GQIR, and DWT-EFRQI in terms of Peak Signal to Noise Ratio (PSNR) and bit rate.

Keywords: *DCTEFRQI, EFRQI, DCT, DCT-GQIR, DWT-GQIR, DWT-EFRQI, (PSNR)*

INTRODUCTION

In the realm of quantum computer science, physics and mathematics play a crucial role in advancing the field of quantum information processing (QIP) 1. In the quantum domain, Hilbert space provides ample room to map quantum information. Quantum mechanics deals primarily with the quantum properties in a quantum system, with entanglement and superposition being two key properties that enable faster computation 2. Due to its remarkable quantum properties and faster computation, quantum computing has gained increasing research interest worldwide. Moreover, quantum parallelism is an inherent phenomenon that makes it unique and faster compared to classical computers [3, 4]. Classical computers have several limitations, including the inability to solve NP-hard problems rapidly, the need for routine pattern recognition without a detailed understanding of the problem subject, slow computational time compared to quantum computers, and the challenge of optimization - finding the best solution to a problem among many possibilities [9, 11].

1. They are not capable of solving NP-hard problems rapidly.

2. Pattern recognition work is routine and does not require a detailed understanding of the problem subject.

3. They have slower computational time compared to quantum computers.

4. Optimization, which involves finding the best solution to a problem among many possibilities, is a challenging task for classical computers.

Moore's Law states that the computing power of classical computers has increased over the past decade, but its growth has slowed down due to several objective factors [5]. As a result, there is a growing demand to increase computing power. Feynman et al. explored the first quantum computer as an alternative way to increase computing power, which gained popularity in the research and development community [6]. Shor proposed an algorithm for factorial calculation for integers in a quantum computer in [7], which demonstrated faster computation compared to classical computers. Grover subsequently provided an algorithm for database search in the quantum domain following Shor's algorithm [8]. The advantages of quantum computers are outlined below [9, 10, 11, 12].

However, quantum computers also have some limitations [9, 10, 11, 12], such as:

- They may not have an advantage over classical computers for some applications, like chess playing, airline flight scheduling and proving theorems.
- They may produce errors due to unwanted interactions with their environment, which is known as decoherence.
- They may only show one outcome among many possibilities when measured.

Quantum computers have the following advantages [9, 10, 11, 12]:

- They can solve NP (non-deterministic polynomial) problems quickly.
- They can process all possible answers simultaneously with their hardware.
- They can compute faster than classical computers.
- They can store huge amounts of information with the exponential formula (2n).
- They can generate mathematical creativity automatically.
- They can speed up cryptographic codes and online monetary transactions.

However, quantum computers also face some challenges [9, 10, 11, 12], such as:

- They may not perform better than classical computers for some applications, like chess playing, airline flight scheduling and proving theorems.
- They may produce errors due to decoherence, which is the unwanted interaction with their environment.
- They may only show one outcome among many possibilities when measured.

When a quantum computer is measured, all other results except for the displayed one will disappear.

- As microchip transistors approach the atomic scale, ideas from quantum computing are likely to become relevant for classical computing as well.

Despite the pros and cons, there are many areas of application and development for quantum computers, as outlined in references [11, 12, 18].

- Quantum computers have many potential applications,
- including identifying patterns in the stock market,
- recording weather or brain activity,
- drug discovery,
- cryptographic systems.
- They can also be used for storing items on shelves at the same time as warp-drive generators and anti-gravity shields,
- chemical simulation,
- radar making, and
- weather forecasting

Applications of quantum computer in term of image processing are recorded below [9, 13]:

- Image compression.
- Image segmentation
- Edge detection.
- Security and denoising of images.
- Information security.
- Watermarking.
- High privacy.
- QCNN for image processing.

The combination of quantum and classical computers is becoming increasingly important in resolving the presentation and processing issues of classical images, as noted in reference [5]. Image processing is a core characteristic of many applications, and classical image processing requires a high number of operations. The use of images is expanding in areas such as image pattern learning, image registration, image sensor data, agriculture, medicine, and remote sensing, as outlined in reference [15]. However, the increasing number of images being processed leads to algorithmic complexity in classical computation. To process these large images, classical computation requires more memory and hardware, [15].

Quantum computers can process and store large amounts of image data with high performance algorithms that use qubits instead of binary bits [16] [17]. They can handle n-bit sequences with O(n) complexity, while classical computers need O(n ∗ 2n). They can also compress images in the quantum domain and reduce the number of image operations. The complexity of quantum systems depends on the number of gates, which also determines the time needed for each operation [19]. Compression is often used to make systems more cost-effective in terms of computing power, storage and communications.

This study compares quantum image representation and compression using classical preparation methods. The image is prepared using Discrete Wavelet Transform (DWT) and Discrete Cosine Transform (DCT) before being presented in the quantum system to investigate each method's capability for preparation. The coefficient values are then quantized to make them feasible for representation and compression in the quantum circuit. The proposed algorithm is used to represent the quantized coefficient in the quantum computer. Before presenting in the quantum computer, the quantized coefficient and its corresponding position are prepared to make it appropriate for quantum representation and compression.

This work makes several contributions, including the ability to represent and compress images of various sizes in a quantum computer.

- The complexity of preparation is reduced by applying a pre-preparation approach.
- The quantum bit rate can be calculated for any type of image, and the image can be retrieved from its quantum circuit, including both pixel and position.
- Our approach allows for the inclusion of any type of quantum operation.
- Although things are happening in the quantum computer, resource calculation is done through a classical computer.
- The combination of applications can bring real-life quantum image applications to fruition.

The article is structured as follows. Section 2 reviews the related literature; section 3 describes the proposed method; section 4 reports and discusses the results. Section 5 concludes the work.

2 LITERATURE AND ITS SUMMARY

Quantum computing is a new paradigm that combines quantum mechanics, computer science and mathematics to overcome the limitations of Moore’s law for computational power [5]. Image presentation and image retrieval are the main challenges in quantum image processing. Qubit lattice was the first quantum method to store and retrieve images using a multi-particle quantum system [17]. Latorre et al. developed Real Ket, an algorithm to represent quantum images [20].

FRQI (Flexible Representation of Quantum Image) was inspired by pixel reorientation in classical computers and used angles to represent gray scale images in quantum systems [21]. However, it could not handle complex pixel-wise operations with one qubit. The Entanglement image representation used entanglement to improve the representation [22]. Jiang et al. proposed a quantum compression method based on JPEG (Joint Photography Expert Group) images using DCT (Discrete Cosine Transform) and GQIR (General Quantum Image Representation) [23]. Laurel et al. showed how to convert bit pixel images to quantum pixel images [24]. NEQR (Novel Quantum Representation of Color Digital Images) represented color images in quantum systems with multiple qubits [25]. However, it only worked for square images. INEQR solved this problem by allowing unequal horizontal and vertical lengths [16]. However, it was unclear how it represented large and colorful images.

GQIR (General Quantum Image Representation) can represent rectangular images of any shape with a logarithmic scale, but it creates many redundant bits when preparing the positions [26]. For example, an 2X2 image and its GQIR representation are shown in Table 1.

Table 1: Example of an 2X2 image and its GQIR representation.

79	79
79	79

Figure 1 shows the corresponding 2x2 GQRI quantum circuit representation, which directly converts pixel values and their positions into a quantum circuit. However, to generate each pixel position location, a similar amount of bits is required each time to connect each CNOT gate for preparing the pixel value with its state position, which is the main drawback of this method. If there are more CNOT gates to create each pixel value, it leads to the complexity of state bit preparation.

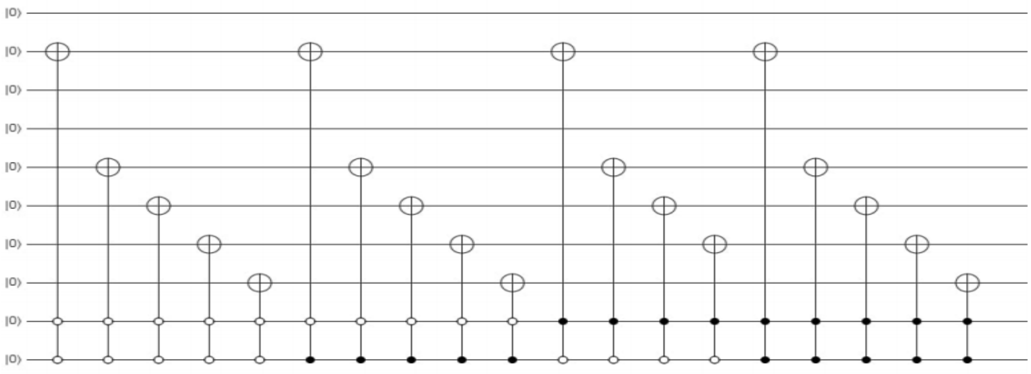


Figure 1: Quantum circuit for the GQIR representation of a 2X2 gray scale image

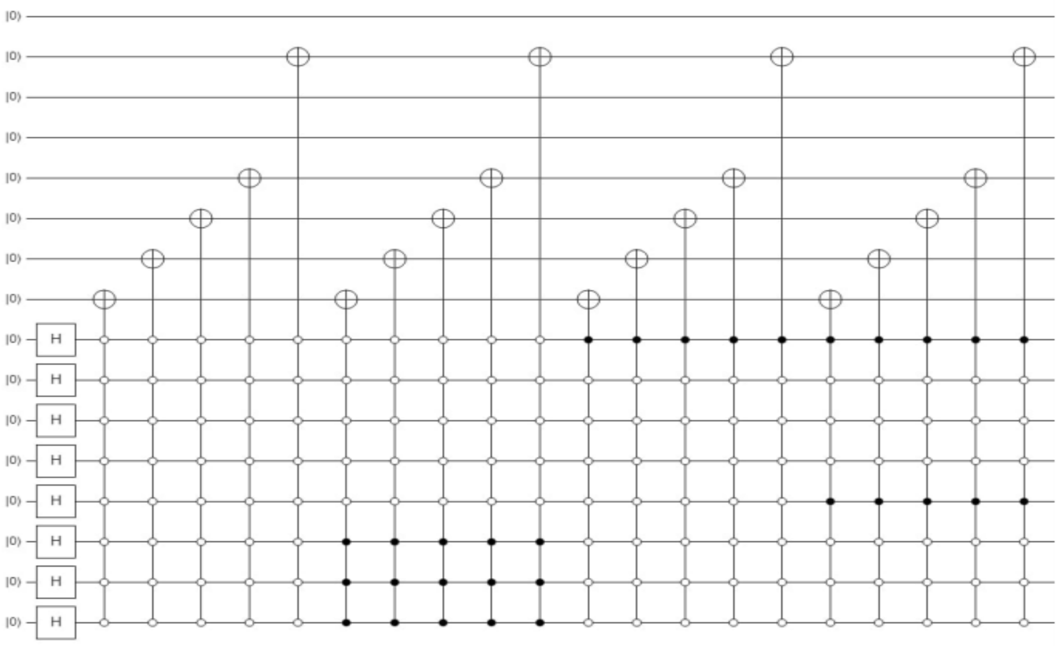


Figure 2: An 16x16 GQIR-DCT quantum image and its quantum state

Fig. 2 shows an example of a 16x16 lena image quantum circuit after 16 quantization using Quirk simulation tool [11]. To map the nonzero coefficient values to quantum states, they are converted to binary streams and only the ones are represented with 8 qubits at the top of the circuit in Fig. 2. To prepare the states, the positions of the nonzero coefficients are also recorded and located in a two-dimensional (YX) system with 8 qubits at the bottom of Fig. 2.

QUALPI (Quantum Angle-based Luminance Polarization Image) was a method to represent quantum images with polar coordinates [27]. NASS (Normal Arbitrary Superposition State) was a multidimensional representation of color images [15]. EFRQI (Efficient Flexible Representations of Quantum Image) was a method to reduce the state preparation bits of GQIR [15].

- Rather than decreasing quantum resources its increase the number of require gate compare to GQIR.
- provides too much complexity to represent every pixel of a medium or big size image such as 512X512 and 1024X1024 respectively.

3 PROPOSED METHODOLOGY

This section explains the proposed research method. Fig. 3 compares our scheme with the traditional one. The traditional scheme represents pixel values directly in quantum computers. Our scheme uses coefficients instead of pixel values, which are obtained by classical preparation. The quantized coefficients and their positions are then prepared for quantum representation. The quantum computer handles the representation, but the classical computer does the calculation. We use 8 and 64 block sizes for DWT and 8 block size for DCT, and Q=8, 16, 32, 36 and 70 quantization factors to evaluate our scheme against the traditional one.

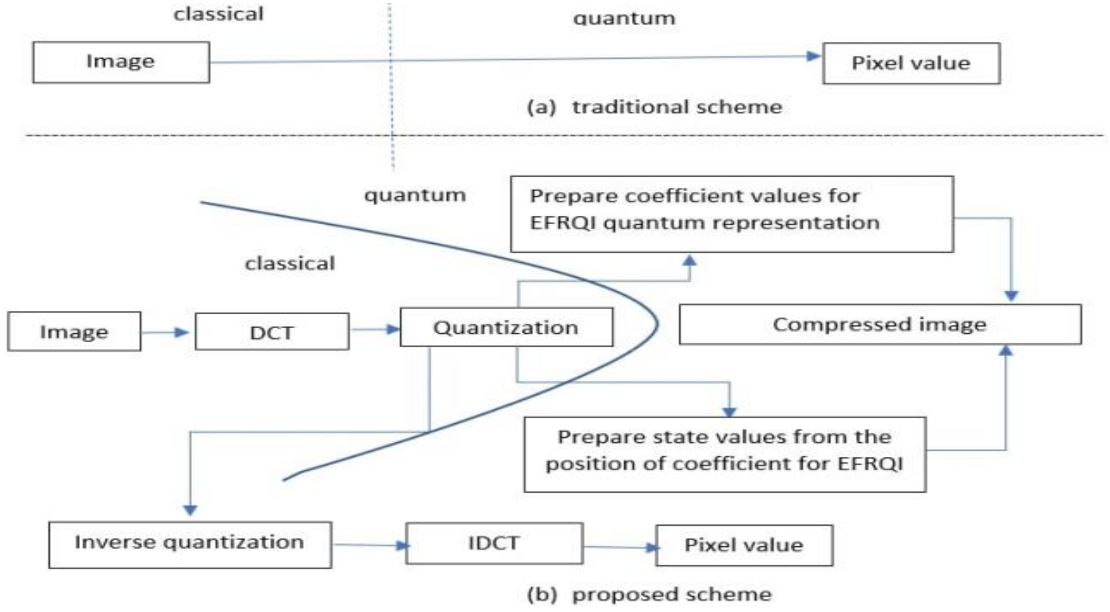
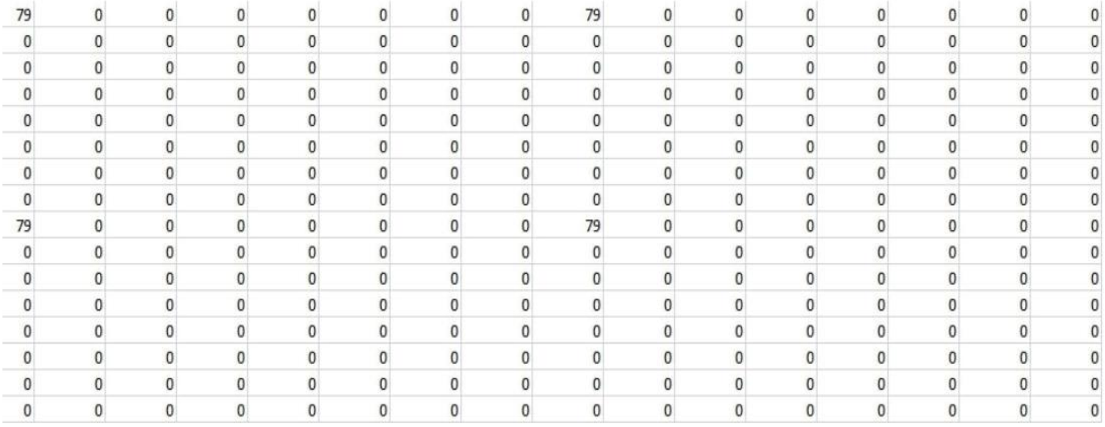


Figure 3: Basic idea of proposed scheme and difference with traditional scheme.

Fig. 4 shows the lena image after DCT preparation. The nonzero coefficients are converted to binary and counted for the number of ones. The quantum circuit uses qubits to represent ones and zeros. The positions of the coefficients are also used to prepare the quantum states. There are two bit rates: one for the coefficients and one for their positions. Fig. 5 shows the quantum image of a 16X16 lena image with the proposed DCTEFRQI method. The complexity of preparation is reduced by compressing with DCT instead of representing pixel values directly in the quantum computer. The coefficients are represented in blocks in the quantum system.



79	0	0	0	0	0	0	0	79	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79	0	0	0	0	0	0	0	79	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 4: Example of Lena image after DCT with 16 label of quantization

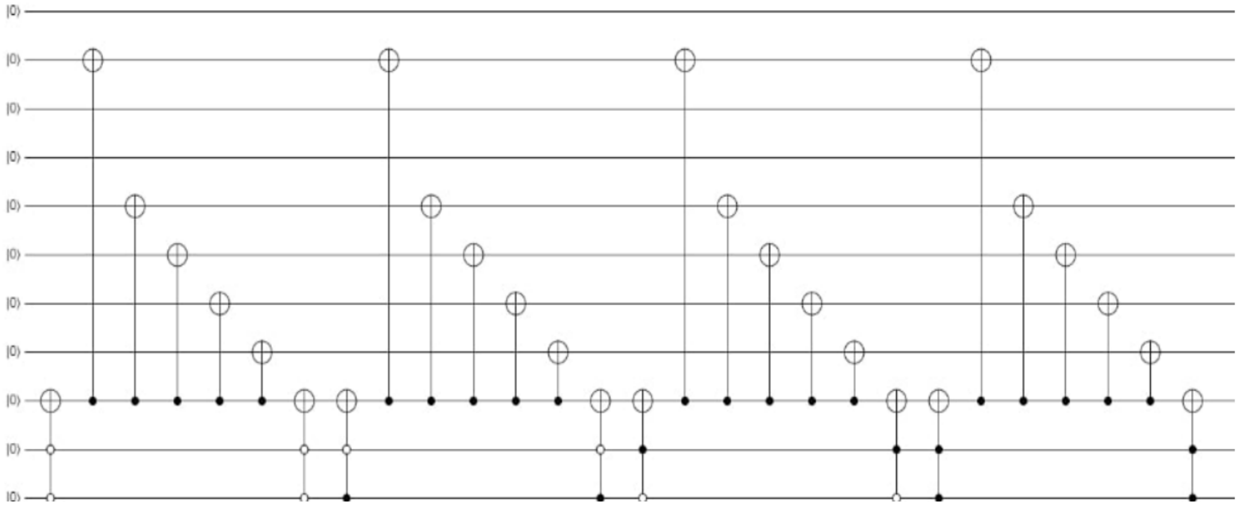


Figure 5: An 16x16 DCTEFRQI quantum image and its quantum state

represented in quantum computer ease the complexity of image preparation before representing in quantum system. $2^n X 2^n$ image size is considered for representation and compression purpose.

In DCTEFRQI approach, steps involve are given below:

Step 1: Apply DCT and quantization.

Step 2: Prepare the quantized DCT coefficients for quantum representation. This is similar to GQIR. DCTEFRQI needs q + 2n + 1 qubits and sets them all to |0>. q is the number of qubits for the nonzero coefficients and depends on the maximum pixel value of the gray image. n is the number of qubits for the image size and is calculated as n = log2(S), where S is the square image size. One auxiliary qubit connects the coefficient qubits and the position qubits. The initial state is given by the following equation [15]:

$$\Psi_0\rangle = |0\rangle^{\otimes(q+2n+1)} \qquad (1)$$

Then, (q + 1) identity gates and 2n Hadamard gates are used for coefficient preparation and its state preparation respectively. The identity and Hadamard matrix are shown below:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (2)$$

$$H = \begin{pmatrix} 1/\sqrt(2) & 1/\sqrt(2) \\ 1/\sqrt(2) & -1/\sqrt(2) \end{pmatrix} \qquad (3)$$

In this step, the whole quantum step can be expressed as follows:

$$U = I^{\otimes q+1} \otimes H^{\otimes 2n} \qquad (4)$$

U transform Ψ_0 from initial state to intermediate state ψ_1.

$$\Psi_1 = U(|\Psi_0\rangle) = I|0\rangle^{\otimes q+1} \otimes H^{\otimes 2n} \tag{5}$$

The final preparation step is done using U_2 quantum operator:

$$\Psi_2 = U_2(|\Psi_1\rangle) = \frac{1}{2^n} \sum_{i=1} \sum^{j=1} |C_{YX}\rangle |YX\rangle \tag{6}$$

where $|CY\ X\rangle$ is corresponding coefficient value and Y X its coordinate position. The quantum transform operator is U_2 is given below:

$$U_2 = \prod_{X=0,\ldots,2^n-1} \prod_{Y=0,\ldots,2^n-1} U_{YX} \tag{7}$$

The quantum sub-operator U_{YX} is also given below:

$$U_{YX} = \left(I \otimes \sum_{ij \neq YX} |ji\rangle\langle ji| \right) + \sigma_{YX} \otimes |YX\rangle\langle YX| \tag{8}$$

The σ_{YX} is given below:

$$\sigma_{YX} = \otimes_{i=0}^{q-1} \sigma_{YX}^{i} \tag{9}$$

The function of σ^{i}_{YX} is setting the vallue of i_{th} qubit of (YX)’s quantized DCT coefficient.

To prepare the state, the Hadamard gate is used to create superposition, and the CNOT gate is used to create entanglement between qubits in the quantum circuit. The identity gates do not have any effect on the qubit's initial state, meaning that the original state of the qubit remains unchanged. The Hadamard gate creates a superposition of the states |0> and |1> with equal probability.

Step 3: after performing an 8x8 block DCT, the quantized coefficient is stored. Then, the non-zero quantized coefficient is prepared for representation in the quantum system. At the same time, the corresponding non-zero coefficient position is recorded for preparing its quantum state. The bit rate is calculated from the non-zero coefficient, which includes only ones. Additionally, when calculating its position, both the frequency of zeros and ones that occur in state connection are considered, along with an extra bit to locate the block position that minimizes the block position error. The sign bit is also considered to assign each coefficient a sign value.

In addition, when calculate it’s position count both frequent number of zero’s and one’s happen in state connection with considering extra bit to locate the block position that minimize the block position error. In addition, sign bit also considered to assign each coefficient sign value.

Step 4: Inverse quantization.

Step 5: Inverse DCT.

Step 6: Compute PSNR to qualify the reconstructed image. The PSNR is defined as follows[23]:

$$PSNR = 20 * log_{10} \frac{MAX1}{\sqrt{MSE}} \tag{10}$$

where MAX1 is the maximum possible pixel value of an image. The MSE(Mean Square Error) is expressed as follows:

$$MSE = \frac{1}{mn} \sum_{0}^{m-1} \sum_{0}^{n-1} ||(i,j) - g(i,j)||^2 \tag{11}$$

Table 2: Selected Sample Image details

Image Name	Image Name
Deer	1024X1024
Cameraman	192X192
Scenery	512X512
Lena	512X512

In the mean time, two times compression already happened through quantum image preparation and presentation. Firstly, compression is happened in the preparation stage and finally its happens again when its represent in quantum circuit since its consider only one’s and discard all zero value to prepare the coefficient.

4 RESULT AND DISCUSSION

Figure 4 shows four types of images selected from a database sample for result verification, and the details of each image are given in Table 2. Figure 6 shows the rate distortion curve for the deer image depicted in Figure 4(a). The results show five different quantization factors, Q=8, 16, 32, 36, and 70, for both DWT-GQIR and DCT-GQIR. In the case of DWT-GQIR, an 8 block size shows better results compared to a 64 block size. In both cases, they exhibit the same PSNR but provide different bit rates, although different combinations of wavelet label 1 were applied. DWT-GQIR shows poorer results than DCT-GQIR over the considered quantization because DWT generates lower values of coefficients in the higher state position, while DCT generates higher values of coefficients in the lower position. The same thing happens over the whole coefficient, which is why DCT-GQIR shows better results compared to DWT-GQIR. The results in Figure 6 show that the proposed DCTEFRQI displays better results compared to DCT-GQIR because DCT-EFRQI is able to create higher coefficient values in lower state positions with the help of an auxiliary qubit and Toffoli gate. The auxiliary qubit was used to connect the coefficient values representing qubits to their corresponding position representing qubits via the Toffoli gate. Both the auxiliary qubit and Toffoli gate contributed to a lower quantum bit rate, meaning that lower quantum operation resources are required in DCTEFRQI compared to DCT-GQIR.

Therefore, DCT-GQIR and DCTEFRQI are considered for the remaining analysis of the results that determine the performance of each combination using the remaining selected sample images (32, 36, and 70). The bit rate makes this difference while exhibiting the same PSNR for DCT-GQIR and DCTEFRQI. It is concluded that DCTEFRQI performs better than DCT-GQIR.

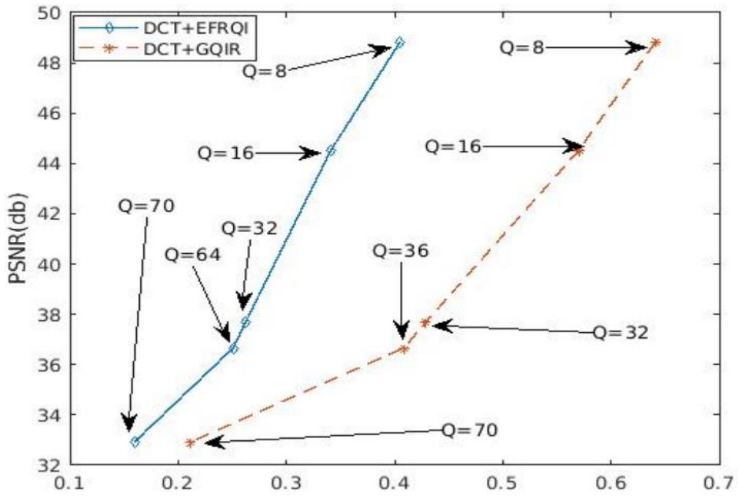


Figure 7: Bit rate versus PSNR for cameraman image

Fig. 8 shows the comparison result of PSNR versus bit rate for scenery image. Result shows that, DCTEFRQI provide better bit rate compare to DCT-GQIR but exhibit same PSNR over considered quantization factor. From this result, It is concluded that, DCTEFRQI enact efficient compression method compare to DCT-GQIR for quantization factor, Q=8, 16, 32, 36, and 70.

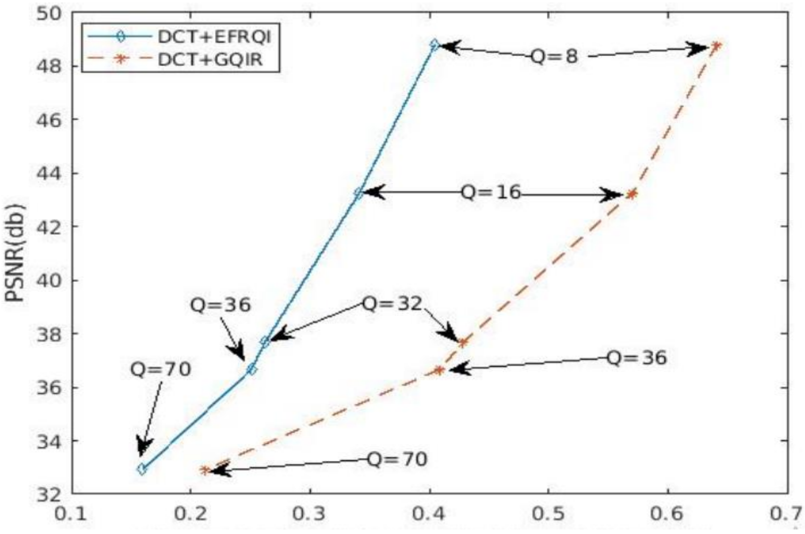


Figure 8: Bit rate versus PSNR for scenery image

Fig. 9 reveals the result of PNSR versus bit rate of lena image for both DCTEFRQI and DCT-GQIR approach. Result shows that, DCT-GQIR provide slightly better result in case of 8, 16 and 32 quantization factor in terms of bit rate.

In contrast, DCTEFRQI performs well compared to DCT-GQIR for the 8 and 70 quantization factors. This can be explained by the fact that more information is lost after applying the quantization factors 16, 32, and 36 in the case of DCT-GQIR compared to DCTEFRQI, while both provide the same PSNR. Based on the region of improvement, it is concluded that DCTEFRQI displays better results compared to the DCT-GQIR approach.

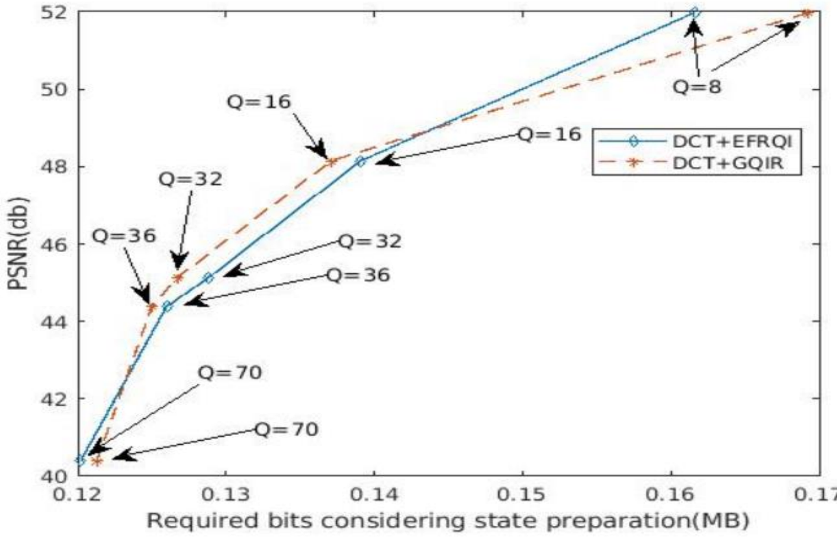


Figure 9: Bit rate versus PSNR for lena image

Figure 10 shows the bit rate comparison results of our proposed method compared to EFRQI. Two types of sample images, a deer image and a Lena image, were considered to determine the bit rate performance. The comparison results show that, in the case of the deer image, DCTEFRQI compresses more than 16 times compared to EFRQI. Additionally, in terms of the Lena image, DCTEFRQI is able to compress more than 44 times compared to EFRQI. Based on these results, it is concluded that DCTEFRQI performs more advanced for bit rate in both 512x512 and 1024x1024 cases of image size.

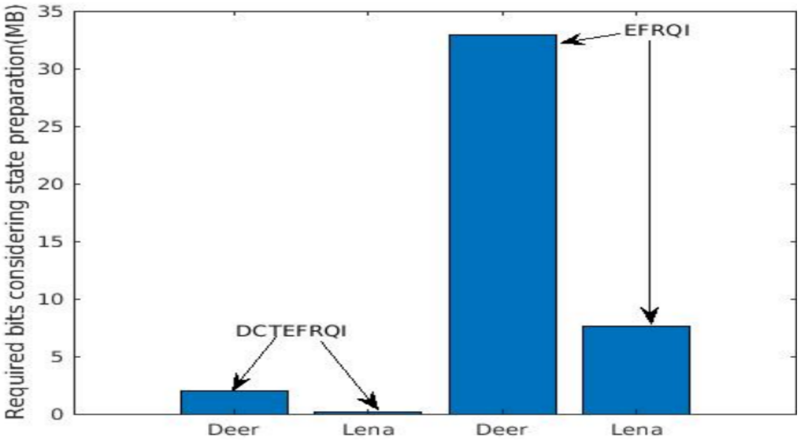


Figure 10: Bit rate comparison for deer and lena image

5 CONCLUSION

This paper focus on the quantum image representation and compression and proposed a new quantum image compression scheme. From this work, the below things are concluded compare to the previous method [23, 18]:

- Any size of image can be represented and compressed in the quantum system.
- Provides better bit rate.

(a) deer image

(b) cameraman image

(c) scenery image **(d) lena image**

Figure 11: Sample image from database [29, 30].

- PNSR do not has any effect if no transform or combination is happened inside quantum system.
- Bit rate is main fact to represent the image inside quantum computer
- Its simple and fast for calculating quantum operation resources.
- Its open up lot of opportunity to processing and compressing of classical image inside quantum processor.
- For medium size image, DCTEFRQI compress more than two times compare to high size image.

Acknowledgment

The author declare that there is no conflict of interest.

REFERENCES

[1] Rabia Amin Khan. An improved flexible representation of quantum images. Quantum Information Processing, vol. 18, no. 7, pages 1-19, 2019.

[2] Jacobs IS and Bean CP. Fine particles, thin films and exchange anisotropy. in Magnetism, pages. 271–350, 1963.

[3] Thaddeus D. Ladd, Fedor Jelezko, Raymond Laflamme, Yasunobu Nakamura, Christopher Monroe, and Jeremy Lloyd O'Brien. Quantum computers. nature, vol. 464, no. 7285, pages. 45-53, 2010.

[4] Mandra Salvatore, Gian Giacomo Guerreschi, and Alán Aspuru-Guzik. Faster than classical quantum algorithm for dense formulas of exact satisfiability and occupation problems. New Journal of Physics, vol. 18, no. 7, pages. 073003, 2016.

[5] Zhaobin Wang, Minzhe Xu, and Yaonan Zhang. Review of quantum image processing. Archives of Computational Methods in Engineering, pages. 1-25, 2021.

Richard P. Feynman. Simulating physics with computers. In Feynman and computation, pp. 133-153. CRC Press, 2018.

[6] Peter W. Shor. Algorithms for quantum computation: discrete logarithms and factoring. In Proceedings 35th annual symposium on foundations of computer science, pp. 124-134. Ieee, 1994.

[7] Grover, Lov K. A fast quantum mechanical algorithm for database search. In Proceedings of the twenty-eighth annual ACM symposium on Theory of computing, pp. 212-219. 1996.

[8] https://www.itrelease.com/2020/10/advantages-and-disadvantages-of-quantum-computers

[9] Mohsen Ali and Mo Tiwari. Image Compression and Classification Using Qubits and Quantum Deep Learning. arXiv preprint arXiv:2110.05476 (2021).

[10] Scott Aaronson. The limits of quantum. Scientific American, vol. 298, no. 3, pp. 62-69.

[11] https://www.forbes.com/sites/forbestechcouncil/2020/09/14/quantum-computing-limits-options-andapplications/?sh=55441ffa5715.

[12] Kalonia, Mohini, and Trasha Gupta Review on applications of quantum image processing. Proc. SPIE 11878, Thirteenth International Conference on Digital Image Processing (ICDIP 2021), 118781T (30 June 2021), https://doi.org/10.1117/12.2601025,

[13] Gonzalez RC and Woods RE. Digital image processing. 2ndEd, Prentice Hall.

[14] Norhan Nasr, Ahmed Younes and Ashraf Elsayed. Efficient representations of digital images on quantum computers. Multimedia Tools and Applications, vol. 80, no. 25, pages.34019-34034, 2021.

[15] Nan Jiang and Luo Wang. Quantum image scaling using nearest neighbor interpolation. Quantum Information Processing,vol. 14, no. 5, pages. 1559-1571, 2015.

[16] Venegas-Andraca, Salvador E., and Sougato Bose. Storing, processing, and retrieving an image using quantum mechanics. In Quantum Information and Computation, Vol. 5105, pages. 137-147, SPIE, Augest, 2003.

[17] Zhang, Yi, Kai Lu, Yinghui Gao, and Mo Wang. NEQR: a novel enhanced quantum representation of digital images. Quantum information processing, vol.12, no. 8, pages. 2833-2860, 2013..

[18] Fei Yan, Abdullah M. Iliyasu, and Salvador E. A survey of quantum image representations. Quantum Information Processing, vol. 15, no. 1, pages. 1-35.

[19] Jose I. Latorre. Image compression and entanglement. arXiv preprint quant-ph/0510031.

[20] Phuc Q. Le, Fangyan Dong, and Kaoru Hirota. A flexible representation of quantum images for polynomial preparation, image compression, and processing operations. Quantum Information Processing, vol. 10, no. 1, pages. 63-84, 2011.

[21] Venegas-Andraca, Salvador E. and Ball JL. Processing images in entangled quantum system. Quant Inf Process, vol. 9, no. 1, pp. 1–11, 2010.

[22] Nan Jiang, Xiaowei Lu, Hao Hu, Yijie Dang and Yongquan Ca. A novel quantum image compression method based on JPEG. International Journal of Theoretical Physics, vol. 57, 3no. pp. 611-636, 2018.

[23] Laurel, Carlos Ortega, Shi-Hai Dong, and M. Cruz-Irisson. Equivalence of a bit pixel image to a quantum pixel image. Communications in Theoretical Physics, vol. 64, no. 5, pages. 501, 2015.

[24] Jianzhi Sang, Shen Wang, and Qiong Li. A novel quantum representation of color digital images. Quantum Information Processing,vol. 16, no. 2, pages. 1-14.

[25] Jiang, Nan, Jian Wang, and Yue Mu. Quantum image scaling up based on nearest-neighbor interpolation with integer scaling ratio. Quant Inf Process, vol. 14, no. 11, pages. 4001–4026.

[26] Zhang, Yi, Kai Lu, Yinghui Gao, and Kai Xu. A novel quantum representation for log-polar images. Quant Inf Process, vol. 12, no.9, pages. 3103–3126.

[27] https://algassert.com/quirk.

[28] https://sipi.usc.edu/database/ (2022).

[29] http://imagedatabase.cs.washington.edu/ (2016).