ON THE ICI SELF-CANCELLATION SCHEMES FOR OFDM SYSTEMS

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ABSTRACT

Intercarrier interference (ICI) is a major problem of orthogonal frequency division multiplexing (OFDM) systems. The difference between transmitter and receiver local oscillator frequencies results ICI in OFDM systems. Self-ICI cancellation is a simple and effective scheme to cancel the effect of ICI. A number of variants of Self-ICI cancellation schemes, like simple self, DFT-based ICI cancellation, conjugate cancellation (CC), and phase rotated conjugate cancellation (PRCC) schemes are studied in this paper. The throughput of all these schemes is same. Simulation results show that the bit error rate (BER) performance of PRCC scheme is much better than the other schemes under high frequency offset environment. However, carrier to interference ratio (CIR) of CC is better than other schemes for low value of frequency offset.

Keywords: Carrier Frequency Offset, Common Phase Error (CPE), ICI Cancellation, And System Throughput.

I. INTRODUCTION

OFDM is known as a bandwidth efficient and very effective modulation technique in multipath fading scenario. One of the major drawback of OFDM system is the problem of carrier frequency offset (CFO), which breaks the orthogonality among subcarriers and causes ICI. ICI affects the system performance severely \cite{1}. A number of schemes to mitigate the effect of ICI using CFO correction at the transmitter \cite{2-8} are available in the literature. A simple and effective method known as ICI self-cancellation scheme is proposed in \cite{2-3}, which uses data repetition in frequency domain to reduce the effect of CFO. These schemes are very simple to implement and are also effective to reduce ICI. However, a major problem with these schemes is limited BER performance due to the CPE generated at the receiver. The schemes in \cite{4-6} can reduce the effect of CPE, but it is unable to provide satisfactory performance at high frequency offset. The CC scheme to mitigate the effect of ICI that employs the standard or normal OFDM signal in its first path and conjugate of the first’s path signal in second path is developed in \cite{7}. In a high frequency offset environment the CC scheme as well the scheme presented in \cite{4-6} cannot guarantee good system performance as described in \cite{8}. A general phase rotated conjugate transmission approach (referred as PRCC) obtained by introducing an artificial phase rotation to both the paths of the conventional CC scheme is presented in \cite{8}. The PRCC scheme achieves better BER performance as compared to all the schemes reported in \cite{2-5} but with additional transmitter complexity. In this paper, we present a comparative study on different self-ICI cancellation schemes. The present analysis shows that the CC scheme has higher CIR as compared to all the schemes only for low values of frequency offset. However, its performance degrades for high frequency offset environment. The PRCC scheme has small CIR as compared to CC scheme for low frequency offset values but it has better BER performance as compared
to all other schemes. The only limitation of PRCC scheme is the requirement of the phase information is used at the receiver and a feed back path (channel) to send this information to the transmitter. Thus the complexity of the PRCC scheme is little bit more as compared to other schemes.

II. SYSTEM MODEL

In the standard OFDM system, a high-speed serial data stream \(a_k \in \{0, 1, \ldots, N-1\}\) produces a length \(N\) signal vector as 
\[
x_n = \sum_{k=0}^{N-1} a_k e^{j2\pi nk/N}; \quad n = 0,1,\ldots,N-1
\]
where \(N\) is total number of subcarriers and \(a_k\) is the transmitted data symbol on the \(k\)th subcarrier.

The received signal samples, affected by frequency offset \(\Delta f\) (mismatch in frequencies between transmitter and receiver oscillators) and noise, at the receiver are given by
\[
y_n = x_n e^{j2\pi nk/T} + w_n; \quad n = 0,1,\ldots,N-1
\]
where \(T\) is OFDM symbol duration, \(\varepsilon = \Delta f T\) denotes the normalized value of CFO, and \(w_n\) are the samples of additive white Gaussian noise (AWGN). The signal at \(l\)th subcarrier of the given OFDM symbol, after discrete Fourier transform (DFT), is obtained as
\[
Y_l = a_l e^{j2\pi kl/N} + \sum_{k=0,k\neq l}^{N-1} a_k e^{j2\pi (l-k)N/N} + W_l; \quad l = 0,1,\ldots,N-1
\]
where \(W_l\) is the DFT of noise samples \(w_n\), and \(c(l-k-\varepsilon)\) is usually referred to as ICI weighting coefficient function and can be expressed as
\[
c(l-k-\varepsilon) = \frac{1}{N} \left| \frac{\sin\pi(l-k-\varepsilon)}{\sin\left(\frac{\pi(l-k-\varepsilon)}{N}\right)} \right| \exp\left(\frac{j\pi(l-k-\varepsilon)(1-N)}{N}\right) \]  
\[
= \frac{1}{N} \left| \frac{\sin\pi(l-k-\varepsilon)}{\sin\left(\frac{\pi(l-k-\varepsilon)}{N}\right)} \right| \exp\left(\frac{j\pi(l-k-\varepsilon)(1-N)}{N}\right)
\]

The first term in (3) is the desired information \(a_l\) with amplitude attenuated by a factor \(c(-\varepsilon)\), and the second is regarded as the ICI term.

The CIR of standard OFDM system, for equal energy symbols, is given as
\[
CIR_{std} = \frac{\sum_{l=0}^{N-1} |c(l-\varepsilon)|^2}{\sum_{l=0}^{N-1} |c(k-\varepsilon)|^2}
\]

III. SELF-ICI CANCELLATION SCHEMES

3.1 Self-ICI Cancellation

In the self-ICI cancellation scheme the same data symbol is repeated on the two adjacent subcarriers. From the input data stream \(\{a_k\}_{k=0}^{N-1}\), the \(k\)th data symbol is transmitted on the two adjacent subcarriers as \(\{a_k, -a_k\}\). By doing so, the ICI generated by subcarrier \(l\) is cancelled out automatically with the ICI generated by subcarrier \((l+1)\).
At the receiver, the output of the FFT block is expressed as

\[ Y_l = a_l c(-\varepsilon) + \sum_{k=0, k \neq l}^{N-1} a_k c(l-k-\varepsilon) + W_l; l = 0, 1, ..., N-1 \] (6)

and

\[ Y_{l+1} = \sum_{k=0}^{N-1} a_k c(l+1-k-\varepsilon) + W_{l+1} \] (7)

Since, the \( l \)th carrier is modulated by \( a_l \) and \((l+1)\)th by \(-a_l\). Therefore, the expressions for \( Y_l \) and \( Y_{l+1} \) are modified as

\[ Y_l = \sum_{k=0, k \neq l}^{N-2} a_k \left(c(l-k-\varepsilon) - c(l-k-1-\varepsilon)\right) + W_l \] (8)

and

\[ Y_{l+1} = \sum_{k=0}^{N-2} a_k \left(c(l+1-k-\varepsilon) - c(l-k-\varepsilon)\right) + W_{l+1} \] (9)

To detect the actual data symbols (after avoiding repetition), the received symbols \( Y_l \), \( Y_{l+1} \) must be subtracted in pairs to further reduce ICI.

Thus we have,

\[
\dot{Y}_l = a_l \left[2c(-\varepsilon)-c(-1-\varepsilon)+c(1-\varepsilon)\right] \\
+ \sum_{k=0, k \neq l}^{N-2} a_k \left[2c(l-k-\varepsilon)-c(l-k-1-\varepsilon)+c(l-k+1-\varepsilon)\right] + W_l
\] (10)

From (10), the CIR for the ICI self-cancellation scheme in case of equal symbol energies can be expressed as

\[
CIR_{self} = \frac{\left\|2c(-\varepsilon) - c(-1-\varepsilon) + c(1-\varepsilon)\right\|^2}{\sum_{k=2}^{N-2} \left\|2c(-k-\varepsilon) - c(-k-1-\varepsilon) + c(-k+1-\varepsilon)\right\|^2} \] (11)

### 3.2 DFT-based ICI Cancellation

In this scheme, the same input data symbols are not transmitted on two adjacent subcarriers. The input data stream is divided into two equal parts as

\[
a^{(1)} = (a_0, a_1, ..., a_{N/2-1})^T, \ a^{(2)} = (a_{N/2}, a_{N/2+1}, ..., a_{N-1})^T
\]

and then DFT of each part is taken. The DFT is represented as \( A^{(1)} = \text{dft}\{a^{(1)}\} \), \( A^{(2)} = \text{dft}\{a^{(2)}\} \). After that the two DFT output signals are combined and IFFT of whole signal is taken as: \( x_n = \text{ifft}\{A^{(1)}, A^{(2)}\} \).

In the matrix form, the transmitted signal is given by \( x_d = \frac{1}{\sqrt{N}} W_N^H B a = X a \), where elements of \( X \) are given in [5] as
At the receiver, if additive noise is omitted, the received signal is expressed as

$$R_0 = \frac{1}{N} B^H W_N E W^H N B a = X^H E X a$$

(13)

Let $D = X^H E X$, the elements at the $i$th row and $j$th column of $D$ is given in [5]. From these elements it can be shown that the ICI’s of the DFT-based scheme is different than the normal OFDM system. The CIR of this scheme is expressed as

$$CIR_{nt} = \frac{\sum_{k=1}^{N} |d_{i,k}|}{\sum_{k=1}^{N} |d_{i,k}|^2}$$

(14)

The DFT-based ICI cancellation scheme is different from the self-ICI cancellation scheme proposed in [4, 5] in terms of spectral efficiency. The spectral efficiency of DFT-based scheme is doubled as compared to other schemes presented in this paper. To compare the performance of DFT-based scheme with other ICI cancellation schemes, same throughput is maintained in this scheme.

### 3.3 CC Scheme

The conjugate cancellation (CC) scheme employs two-path transmission. In its first path, the standard OFDM signal is transmitted whereas in second path conjugate of first path signal is transmitted. In the presence of $\epsilon$, the received signal of the two transmission paths can be expressed as

$$y_n^{(1)} = x_n \exp \left( \frac{j2\pi n}{N} \right) + w_n^{(1)}$$

and

$$y_n^{(2)} = x_n^* \exp \left( \frac{j2\pi n}{N} \right) + w_n^{(2)}$$

(15)

(16)

where $x_n^*$ is conjugate of $x_n$, $w_n^{(1)}$ and $w_n^{(2)}$ are AWGN samples and $n = 0, 1, ... , N-1$.

At the receiver, $y_n^{(1)}$ and the conjugate of $y_n^{(2)}$ are applied to the DFT block to obtain the frequency domain signal samples. The average of these samples is taken to obtain the required output signal samples.

Ignoring the effect of noise, the carrier to interference ratio (CIR) of the CC scheme for equal symbol energies is given as

$$CIR_{cc} = \frac{\sum_{k=1}^{N} |c(-\epsilon) + c(\epsilon)|^2}{\sum_{k=1}^{N} |c(k+\epsilon) + c(k-\epsilon)|^2}$$

(17)

This scheme improves the carrier to interference ratio (CIR) and BER performance over standard ICI self-cancellation and ACSR scheme for low frequency offset values only. For higher values of frequency offset, it fails to correct the CPE and ICI, and the CIR performance of the scheme significantly degrades.
3.5 PRCC Scheme

The PRCC scheme employs two-path transmission like conventional CC scheme of [5], but introduces a new variable \( \phi \) in both the paths. In this scheme, the transmitted signal is so tuned that the ICI effects of the two paths’ signals mutually cancel at the receiver.

In the presence of CFO, the received signals samples are given as in [6] by

\[
Z_k = \frac{1}{2} \left\{ \sum_{l=0}^{N-1} a_l (e^{j\phi l (k - l + \varepsilon)} + e^{-j\phi l (l - k + \varepsilon)}) + (W_l^{(1)} + W_l^{(2)}) \right\}
\]

where \( c(l - k - \varepsilon) \) is given in (4).

Ignoring the effect of noise, the carrier to interference ratio (CIR) of the general PRCC scheme for equal symbol energies is given as

\[
CIR_{PRCC} = \frac{|e^{j\phi \varepsilon c(\varepsilon)} + e^{-j\phi \varepsilon c(\varepsilon)}|^2}{\sum_{l=0}^{N-1} |e^{j\phi c(k + \varepsilon)} + e^{-j\phi c(k - \varepsilon)}|^2}
\]

The optimum value of phase rotation for a given \( \varepsilon \) is that value, which results in maximum CIR and is derived in [6] as

\[
\phi_{opt} = -\pi \varepsilon (N-1)/N
\]

Thus the frequency offset estimation is performed at the receiver and the information is sent back to the transmitter, where optimum phase rotation is applied.

IV. SIMULATION RESULTS

To judge the performance of the all these schemes a number of computer simulations were performed. The standard OFDM system, the ICI self-cancellation scheme, the DFT-based ICI cancellation scheme, the CC scheme, and the PRCC scheme are simulated using MATLAB for AWGN channels. The derivations in Section II – Section III are obtained for AWGN channel. The OFDM system with 64 subcarriers (i.e. \( N = 64 \)) with PSK data symbols is considered. The carrier frequency offset \( \varepsilon \) is chosen as 0.1, 0.2 and 0.3.

The BER performance of the DFT-based ICI cancellation scheme, CC scheme, ICI self-cancellation scheme, and PRCC scheme at different values of CFO is plotted in Fig. 1- Fig. 3 for AWGN channels. From Fig.1- Fig. 3, it is evident that the BER of the PRCC scheme is at small value of frequency offset \( \varepsilon = 0.1 \) is comparable to CC scheme. Furthermore, if \( \varepsilon \) is large \( \varepsilon = 0.2, \varepsilon = 0.3 \), the BER of the PRCC scheme is better than that of all other schemes.

The BER performance of all these systems is also verified under fading environment as well. The fading channel is modeled with the six paths Typical Urban (TU) delay profile [9]. The results are obtained under the condition of perfect channel equalization as considered in [8]. The same BER performance of these schemes under fading channel is also obtained.

It has also been verified through simulations that the PRCC scheme performs better than other schemes under higher order signal constellation (i.e. for 16-PSK). This is also observed for \( \varepsilon = 0.3 \) and \( N > 64 \).
Fig. 1. BER versus SNR for all schemes under AWGN channel at $\varepsilon = 0.1$.

Fig. 2. BER versus SNR for all schemes under AWGN channel at $\varepsilon = 0.2$.

Fig. 3. BER versus SNR for all schemes under AWGN channel at $\varepsilon = 0.3$. 
V. CONCLUSION

A comparative study on ICI self-cancellation schemes for OFDM systems is presented in this paper. It is observed that the PRCC scheme that offers better BER performance than the other self-ICI cancellation schemes at high frequency offset conditions. However, in this scheme a feedback path is required to sent the estimated value of frequency offset to the transmitter. The DFT-based ICI cancellation scheme is spectral efficient as compared to all other self-ICI cancellation schemes but its BER performance is poor as that of PRCC schemes.

REFERENCES